MAGNETIC EFFECT OF CURRENT - I

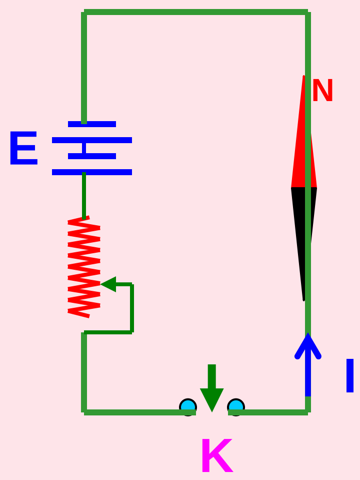
1. Magnetic Effect of Current – Oersted's Experiment
2. Ampere's Swimming Rule
3. Maxwell's Cork Screw Rule
4. Right Hand Thumb Rule
5. Biot – Savart's Law
6. Magnetic Field due to Infinitely Long Straight Current – carrying Conductor
7. Magnetic Field due to a Circular Loop carrying current
8. Magnetic Field due to a Solenoid

Magnetic Effect of Current:

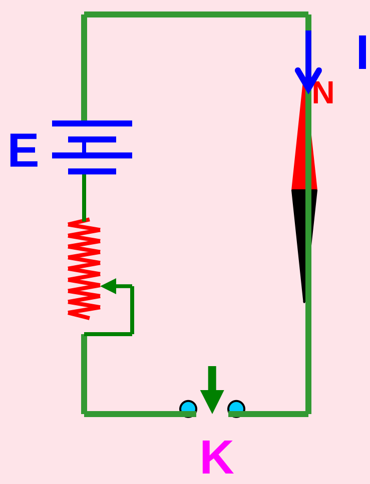
An electric current (i.e. flow of electric charge) produces magnetic effect in the space around the conductor called strength of Magnetic field or simply Magnetic field.

Oersted's Experiment:

When current was allowed to flow through a wire placed parallel to the axis of a magnetic needle kept directly below the wire, the needle was found to deflect from its normal position.



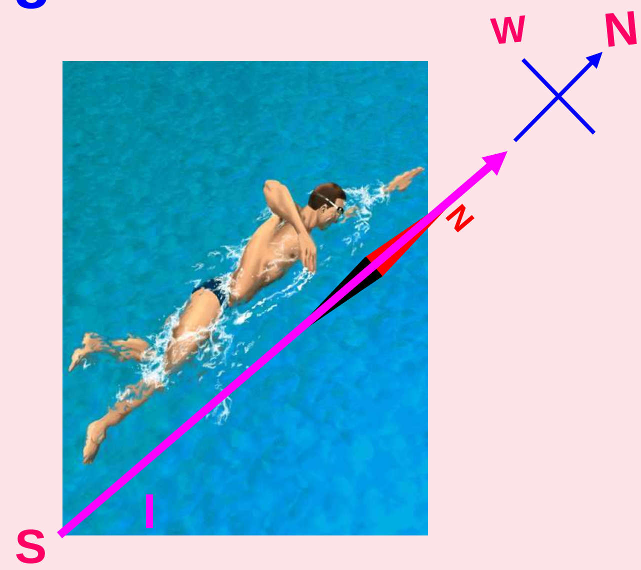
When current was reversed through the wire, the needle was found to deflect in the opposite direction to the earlier case.



Rules to determine the direction of magnetic field:

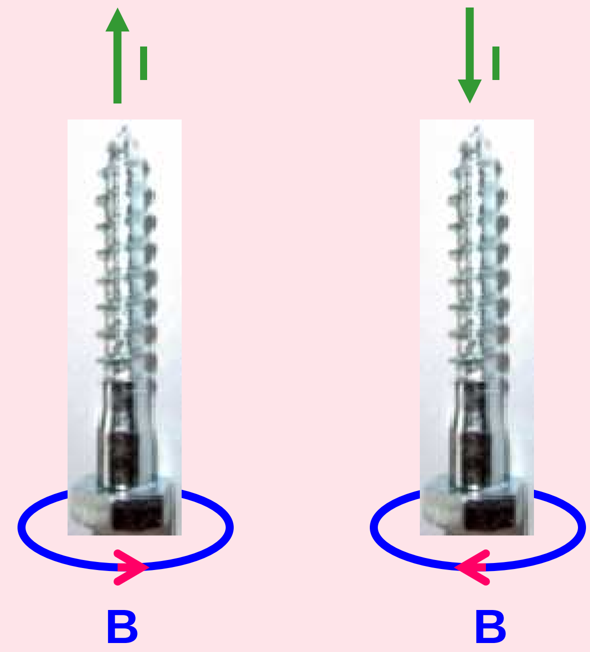
Ampere's Swimming Rule:

Imagining a man who swims in the direction of current from south to north facing a magnetic needle kept under him such that current enters his feet then the North pole of the needle will deflect towards his left hand, i.e. towards West.



Maxwell's Cork Screw Rule or Right Hand Screw Rule:

If the forward motion of an imaginary right handed screw is in the direction of the current through a linear conductor, then the direction of rotation of the screw gives the direction of the magnetic lines of force around the conductor.



Right Hand Thumb Rule or Curl Rule:

If a current carrying conductor is imagined to be held in the right hand such that the thumb points in the direction of the current, then the tips of the fingers encircling the conductor will give the direction of the magnetic lines of force.

Biot – Savart's Law:

The strength of magnetic field dB due to a small current element dl carrying a current I at a point P distant r from the element is directly proportional to I, dl, sin θ and inversely proportional to the square of the distance (r^2) where θ is the angle between dl and r.

i) $dB \propto I$

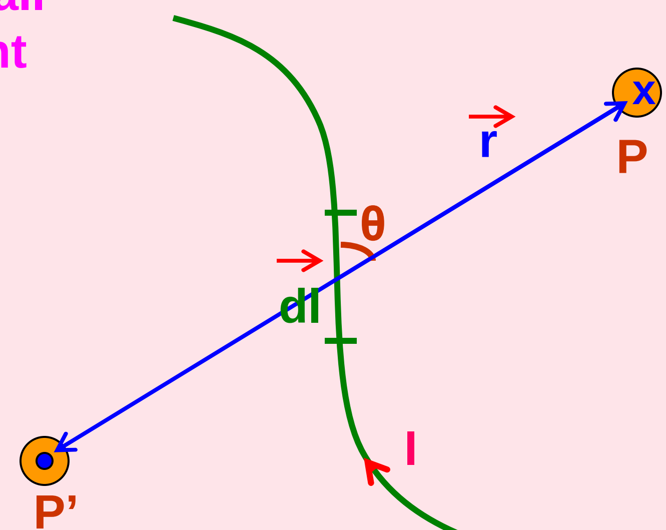
ii) $dB \propto dl$

iii) $dB \propto \sin\theta$

iv) $dB \propto 1 / r^2$

$$dB \propto \frac{I\, dl \sin\theta}{r^2}$$

$$\boxed{dB = \frac{\mu_0}{4\pi}\frac{I\, dl \sin\theta}{r^2}}$$



Biot – Savart's Law in vector form:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \hat{r}}{r^2}$$

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \vec{r}}{r^3}}$$

Value of $\mu_0 = 4\pi \times 10^{-7}$ Tm A^{-1} or Wb m^{-1} A^{-1}

Direction of $d\vec{B}$ is same as that of direction of $d\vec{l} \times \vec{r}$ which can be determined by Right Hand Screw Rule.

It is emerging ⊙ at P' and entering ⊗ at P into the plane of the diagram.

Current element is a vector quantity whose magnitude is the vector product of current and length of small element having the direction of the flow of current. ($I\, d\vec{l}$)

Magnetic Field due to a Straight Wire carrying current:

According to Biot – Savart's law

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl \sin\theta}{r^2}$$

$\sin\theta = a / r \;\; = \cos\Phi$

or $\quad r = a / \cos\Phi$

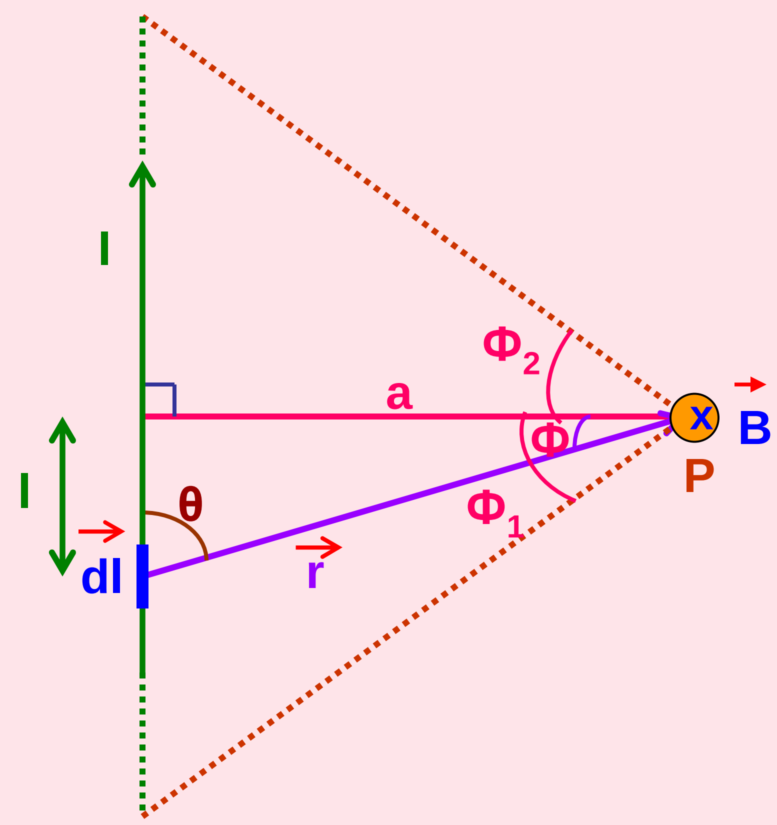
$\tan\Phi = l / a$

or $\quad l = a \tan\Phi$

$dl = a \sec^2\Phi \, d\Phi$

Substituting for r and dl in dB,

$$dB = \frac{\mu_0}{4\pi} \frac{I \cos\Phi \, d\Phi}{a}$$



Magnetic field due to whole conductor is obtained by integrating with limits $-\Phi_1$ to Φ_2. (Φ_1 is taken negative since it is anticlockwise)

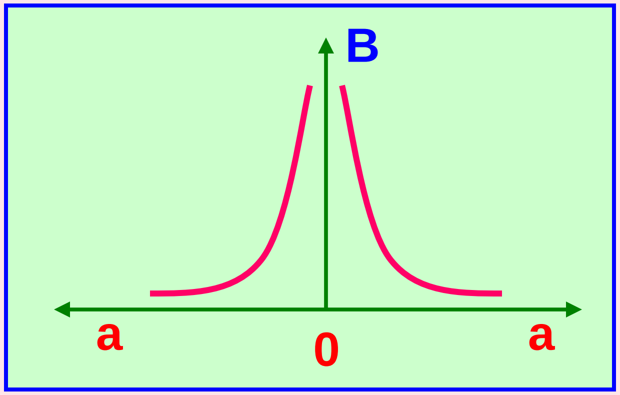
$$B = \int dB = \int_{-\Phi_1}^{\Phi_2} \frac{\mu_0}{4\pi} \frac{I \cos\Phi \, d\Phi}{a}$$

$$B = \frac{\mu_0 I (\sin\Phi_1 + \sin\Phi_2)}{4\pi a}$$

If the straight wire is infinitely long,

then $\Phi_1 = \Phi_2 = \pi / 2$

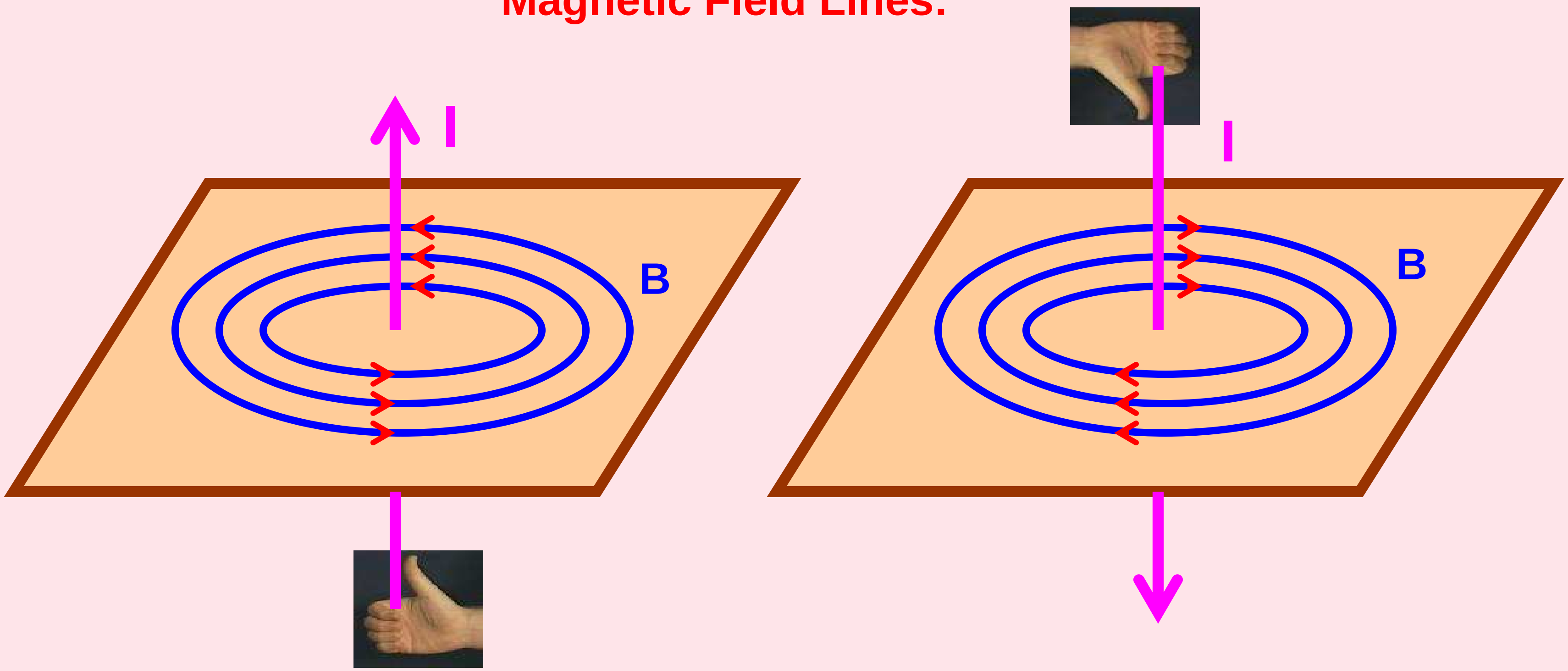
$$B = \frac{\mu_0 \, 2I}{4\pi a} \quad \text{or} \quad B = \frac{\mu_0 \, I}{2\pi a}$$



Direction of $\vec{B}$ is same as that of direction of $\vec{dl} \times \vec{r}$ which can be determined by Right Hand Screw Rule.

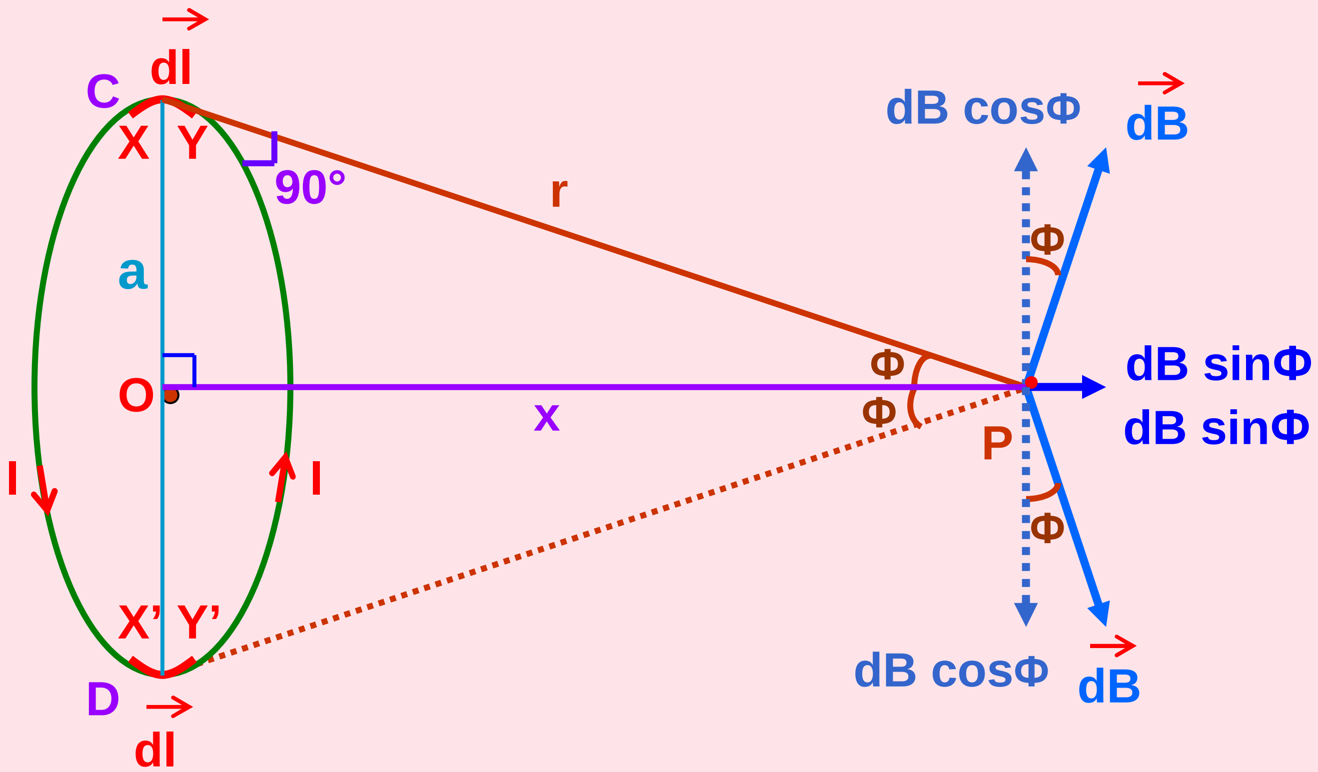
It is perpendicular to the plane of the diagram and entering into the plane at P.

Magnetic Field Lines:



Magnetic Field due to a Circular Loop carrying current:

1) At a point on the axial line:



The plane of the coil is considered perpendicular to the plane of the diagram such that the direction of magnetic field can be visualized on the plane of the diagram.

At C and D current elements XY and X'Y' are considered such that current at C emerges out and at D enters into the plane of the diagram.

$$dB = \frac{\mu_0 \; I \, dl \sin\theta}{4\pi \quad r^2} \quad \text{or} \quad dB = \frac{\mu_0 \; I \, dl}{4\pi \; r^2}$$

The angle θ between $\vec{dl}$ and $\vec{r}$ is 90° because the radius of the loop is very small and since sin 90°= 1

The semi-vertical angle made by $\vec{r}$ to the loop is Φ and the angle between $\vec{r}$ and $\vec{dB}$ is 90° . Therefore, the angle between vertical axis and $\vec{dB}$ is also Φ.

$\vec{dB}$ is resolved into components dB cosΦ and dB sinΦ .

Due to diametrically opposite current elements, cosΦ components are always opposite to each other and hence they cancel out each other.

SinΦ components due to all current elements $\vec{dl}$ get added up along the same direction (in the direction away from the loop).

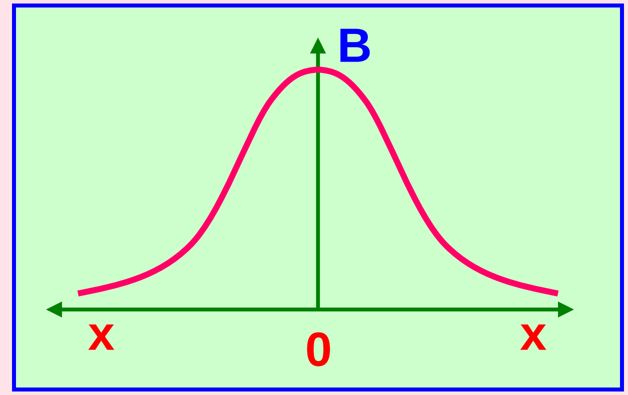
$$B = \int dB \sin\Phi = \int \frac{\mu_0 \; I \, dl \sin\Phi}{4\pi \; r^2} \quad \text{or} \quad B = \frac{\mu_0 \, I \, (2\pi a) \, a}{4\pi \, (a^2 + x^2) \, (a^2 + x^2)^{1/2}}$$

$$\boxed{B = \frac{\mu_0 \, I \, a^2}{2(a^2 + x^2)^{3/2}}}$$

(μ_0 , I, a, sinΦ are constants, $\int dl = 2\pi a$ and r & sinΦ are replaced with measurable and constant values.)

Special Cases:

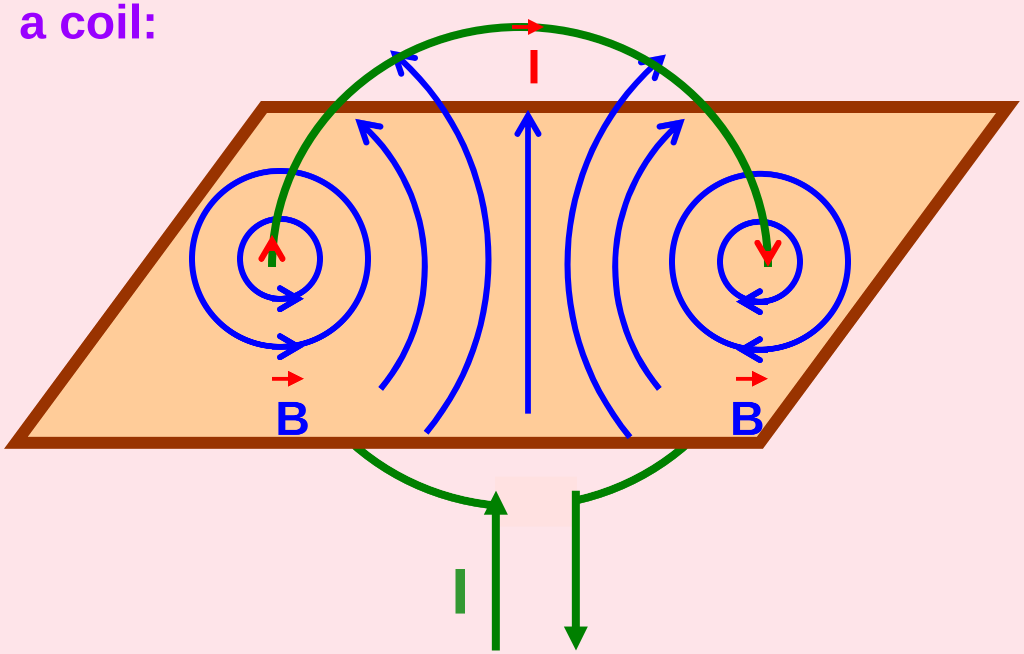
i) At the centre O, x = 0. $\therefore$ $B = \dfrac{\mu_0 I}{2a}$

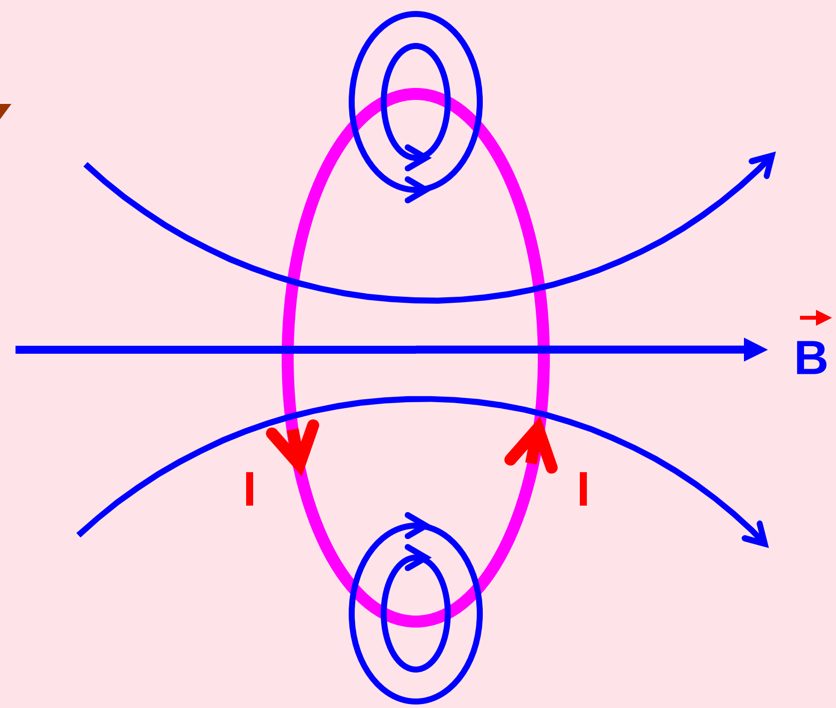


ii) If the observation point is far away from the coil, then a << x. So, a^2 can be neglected in comparison with x^2.

$\therefore$ $B = \dfrac{\mu_0 I a^2}{2 x^3}$

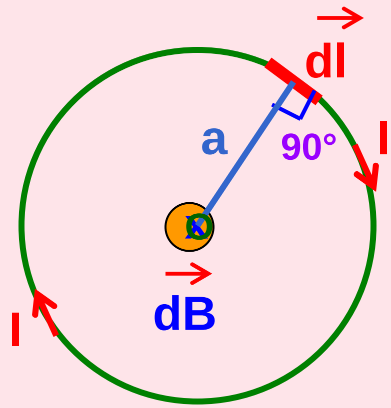
Different views of direction of current and magnetic field due to circular loop of a coil:





2) B at the centre of the loop:

The plane of the coil is lying on the plane of the diagram and the direction of current is clockwise such that the direction of magnetic field is perpendicular and into the plane.



$$dB = \frac{\mu_0 \; I \; dl \sin\theta}{4\pi \quad a^2}$$

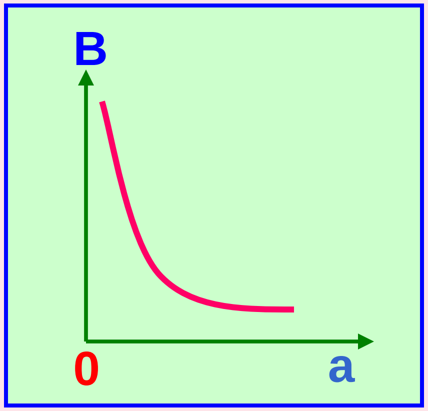
$$dB = \frac{\mu_0 \; I \; dl}{4\pi \; a^2}$$

The angle θ between dl and a is 90° because the radius of the loop is very small and since sin 90°= 1

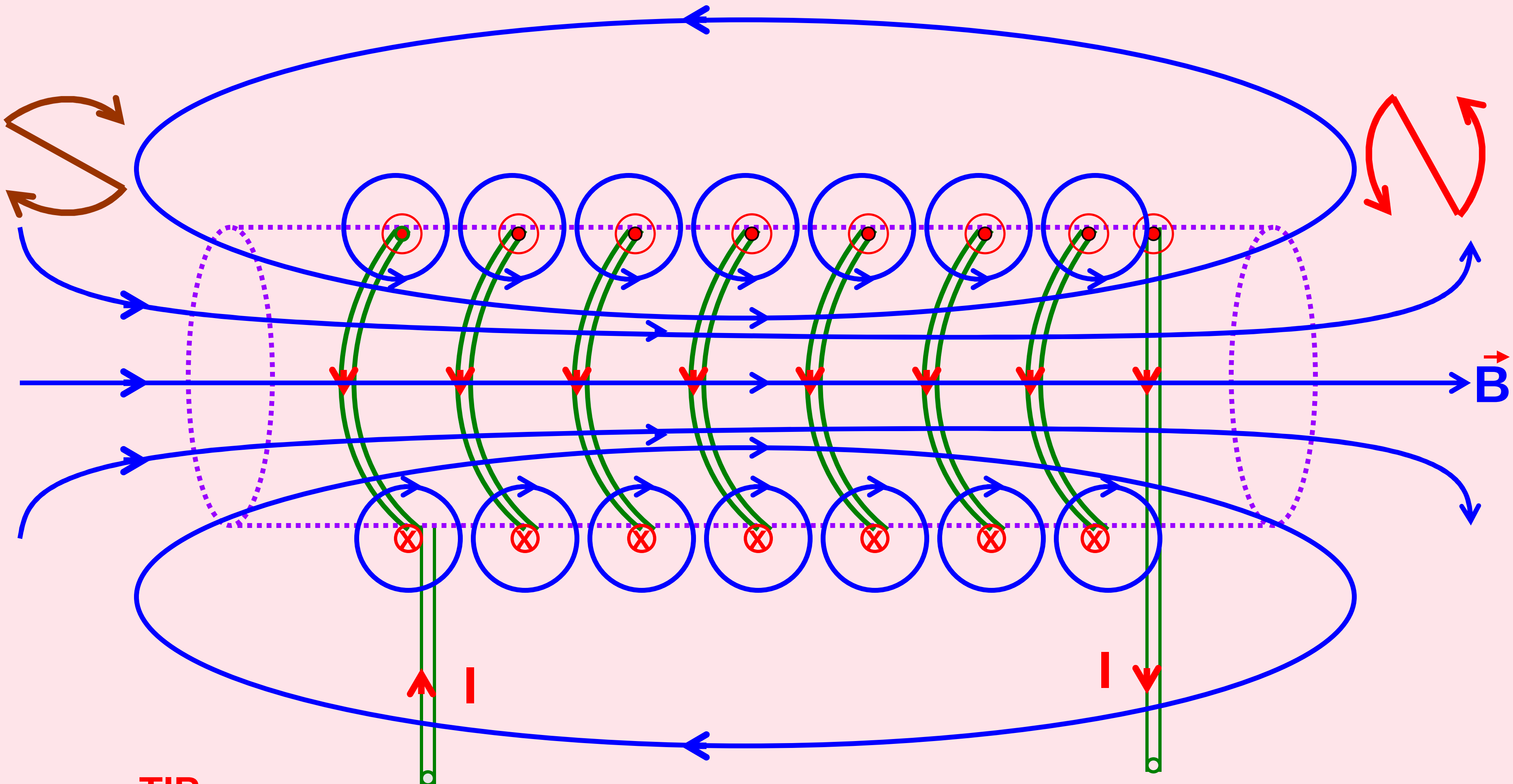
$$B = \int dB = \int \frac{\mu_0 \; I \; dl}{4\pi \; a^2}$$

$$\boxed{B = \frac{\mu_0 \, I}{2a}}$$

(μ_0 , I, a are constants and $\int dl = 2\pi a$)



Magnetic Field due to a Solenoid:



TIP:

When we look at any end of the coil carrying current, if the current is in anti-clockwise direction then that end of coil behaves like North Pole and if the current is in clockwise direction then that end of the coil behaves like South Pole.

MAGNETIC EFFECT OF CURRENT - II

1. Lorentz Magnetic Force
2. Fleming's Left Hand Rule
3. Force on a moving charge in uniform Electric and Magnetic fields
4. Force on a current carrying conductor in a uniform Magnetic Field
5. Force between two infinitely long parallel current-carrying conductors
6. Definition of ampere
7. Representation of fields due to parallel currents
8. Torque experienced by a current-carrying coil in a uniform Magnetic Field
9. Moving Coil Galvanometer
10. Conversion of Galvanometer into Ammeter and Voltmeter
11. Differences between Ammeter and Voltmeter

Lorentz Magnetic Force:

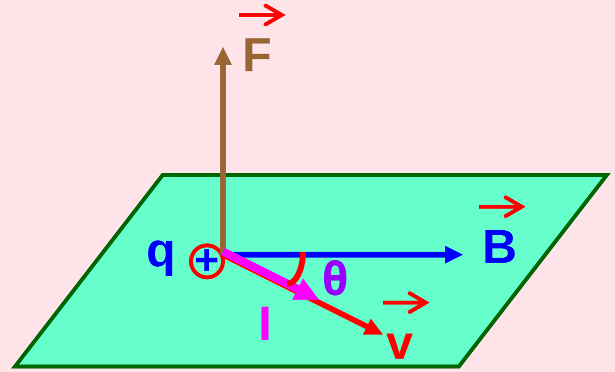
A current carrying conductor placed in a magnetic field experiences a force which means that a moving charge in a magnetic field experiences force.

$$\vec{F}_m = q\,(\vec{v} \times \vec{B})$$

or

$$\vec{F}_m = (q\,v\,B\,\sin\theta)\,\hat{n}$$

where θ is the angle between $\vec{v}$ and $\vec{B}$



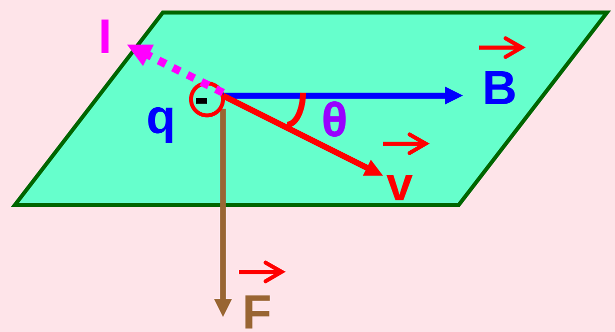
Special Cases:

i) If the charge is at rest, i.e. $v = 0$, then $F_m = 0$.
So, a stationary charge in a magnetic field does not experience any force.

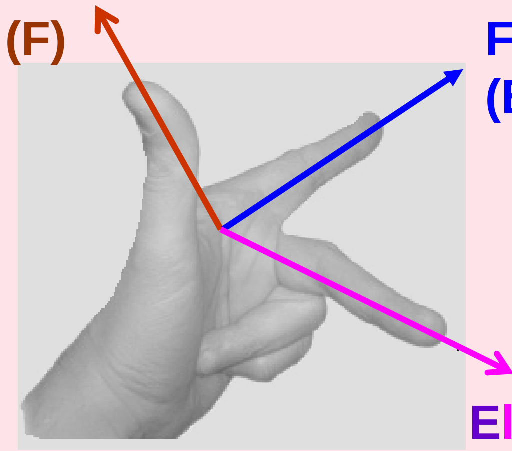
ii) If $\theta = 0°$ or $180°$ i.e. if the charge moves parallel or anti-parallel to the direction of the magnetic field, then $F_m = 0$.

iii) If $\theta = 90°$ i.e. if the charge moves perpendicular to the magnetic field, then the force is maximum.

$$F_{m\,(max)} = q\,v\,B$$



Fleming's Left Hand Rule:



If the central finger, fore finger and thumb of left hand are stretched mutually perpendicular to each other and the central finger points to current, fore finger points to magnetic field, then thumb points in the direction of motion (force) on the current carrying conductor.

TIP:

Remember the phrase 'e m f' to represent electric current, magnetic field and force in anticlockwise direction of the fingers of left hand.

Force on a moving charge in uniform Electric and Magnetic Fields:

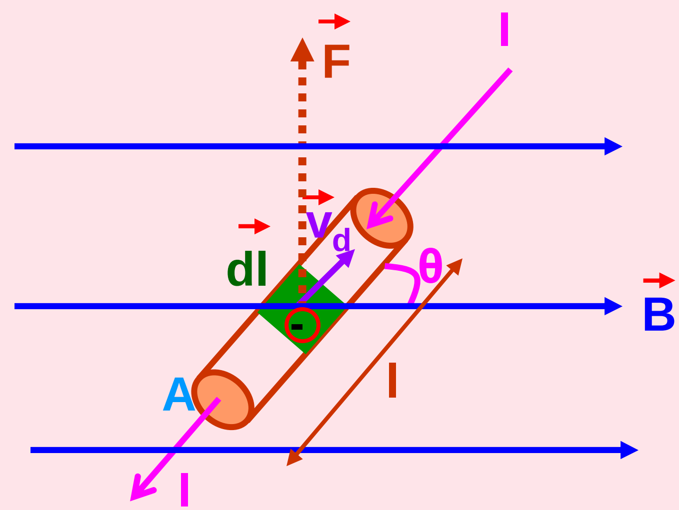
When a charge q moves with velocity $\vec{v}$ in region in which both electric field $\vec{E}$ and magnetic field $\vec{B}$ exist, then the Lorentz force is

$\vec{F} = q\vec{E} + q\,(\vec{v} \times \vec{B})$ or $\vec{F} = q\,(\vec{E} + \vec{v} \times \vec{B})$

Force on a current-carrying conductor in a uniform Magnetic Field:

Force experienced by each electron in the conductor is

$\vec{f} = - e (\vec{v}_d \times \vec{B})$



If n be the number density of electrons, A be the area of cross section of the conductor, then no. of electrons in the element dl is n A dl.

Force experienced by the electrons in dl is

$\vec{dF} = n\,A\,dl\,[- e (\vec{v}_d \times \vec{B})] \;=\; - \;n\,e\,A\,v_d\,(\vec{dl} \times \vec{B})$

$= I\,(\vec{dl} \times \vec{B})$

where $I = neAv_d$ and -ve sign represents that the direction of dl is opposite to that of v_d)

$\vec{F} = \int \vec{dF} = \int I\,(\vec{dl} \times \vec{B})$

$\boxed{\vec{F} = I\,(\vec{l} \times \vec{B})}$ or $\boxed{F = I\,l\,B \sin\theta}$

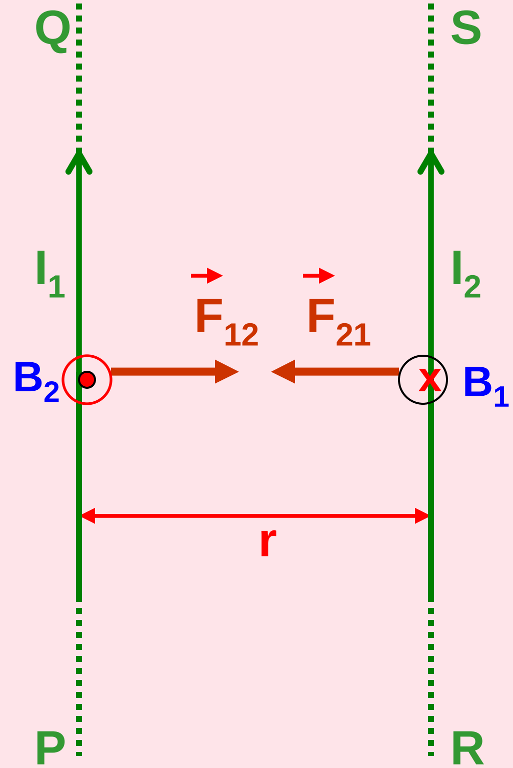
Forces between two parallel infinitely long current-carrying conductors:

Magnetic Field on RS due to current in PQ is

$$B_1 = \frac{\mu_0 I_1}{2\pi r} \quad \text{(in magnitude)}$$

Force acting on RS due to current I_2 through it is

$$F_{21} = \frac{\mu_0 I_1}{2\pi r} I_2 \, l \sin 90^\circ \quad \text{or} \quad F_{21} = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$



B_1 acts perpendicular and into the plane of the diagram by Right Hand Thumb Rule. So, the angle between l and B_1 is 90° .

l is length of the conductor.

Magnetic Field on PQ due to current in RS is

$$B_2 = \frac{\mu_0 I_2}{2\pi r} \quad \text{(in magnitude)}$$

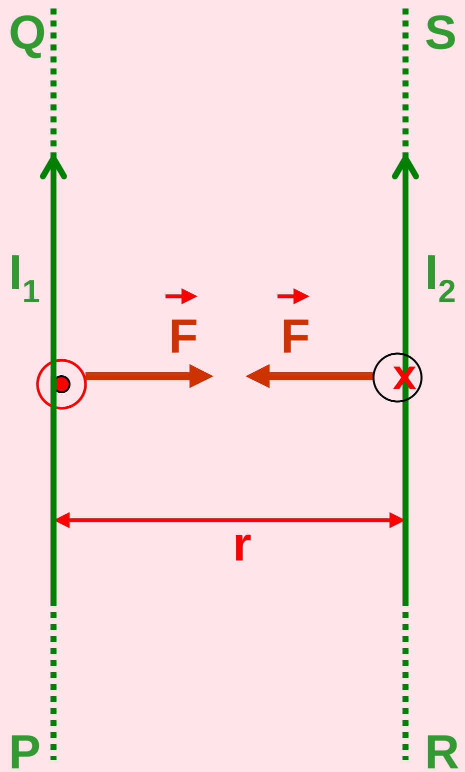
Force acting on PQ due to current I_1 through it is

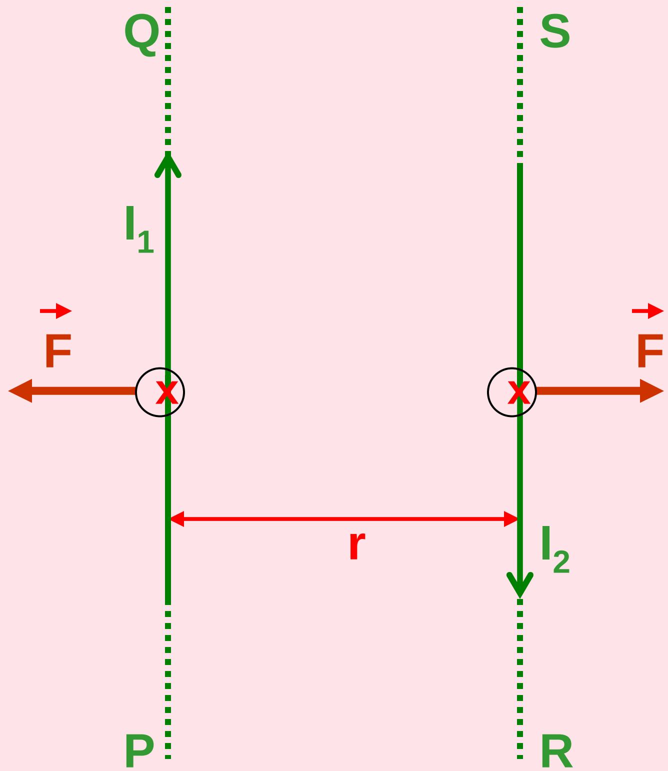
$$F_{12} = \frac{\mu_0 I_2}{2\pi r} I_1 \, l \sin 90^\circ \quad \text{or} \quad F_{12} = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

(The angle between l and B_2 is 90° and B_2 Is emerging out)

$$\boxed{F_{12} = F_{21} = F = \frac{\mu_0 I_1 I_2 l}{2\pi r}}$$

Force per unit length of the conductor is $\boxed{F / l = \frac{\mu_0 I_1 I_2}{2\pi r}}$ N / m





By Fleming's Left Hand Rule, the conductors experience force towards each other and hence attract each other.

By Fleming's Left Hand Rule, the conductors experience force away from each other and hence repel each other.

Definition of Ampere:

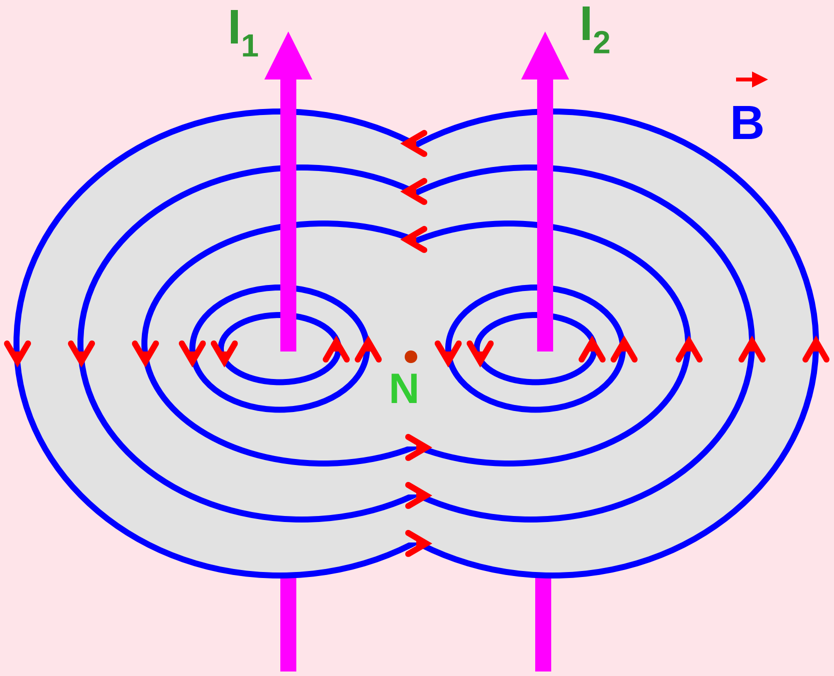
Force per unit length of the conductor is

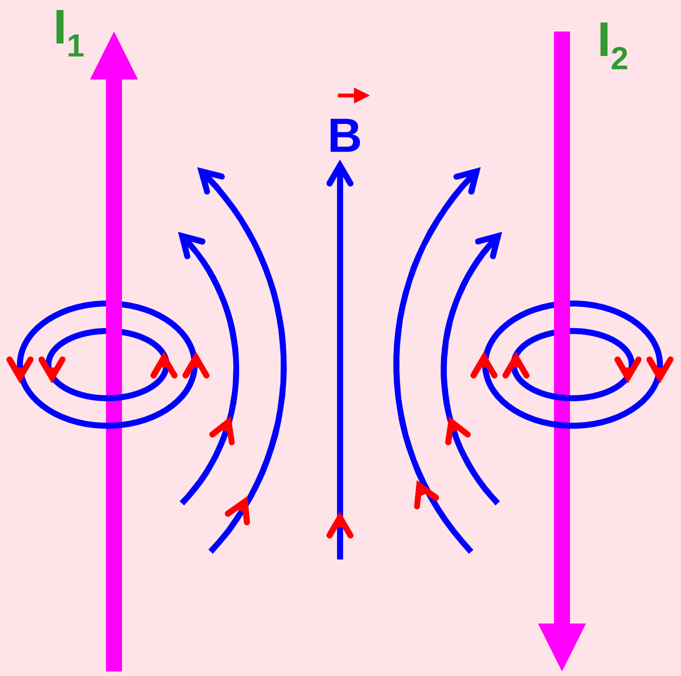
$$F / l = \frac{\mu_0 I_1 I_2}{2\pi r} \quad N / m$$

When $I_1 = I_2 = 1$ Ampere and $r = 1$ m, then $F = 2 \times 10^{-7}$ N/m.

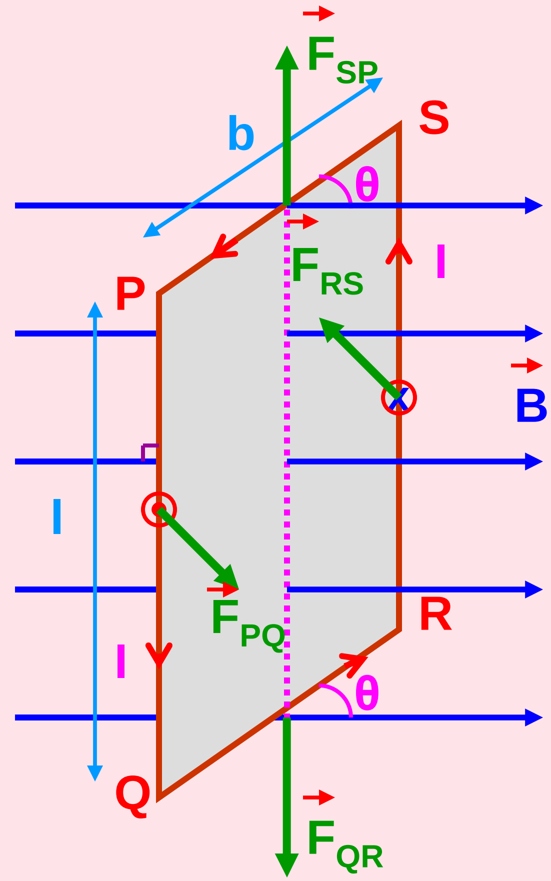
One ampere is that current which, if passed in each of two parallel conductors of infinite length and placed 1 m apart in vacuum causes each conductor to experience a force of 2×10^{-7} Newton per metre of length of the conductor.

Representation of Field due to Parallel Currents:





Torque experienced by a Current Loop (Rectangular) in a uniform Magnetic Field:



Let θ be the angle between the plane of the loop and the direction of the magnetic field. The axis of the coil is perpendicular to the magnetic field.

$$\vec{F}_{SP} = I\,(\vec{b} \times \vec{B})$$

$$|\, F_{SP} \,| = I\, b\, B \sin\theta$$

$$\vec{F}_{QR} = I\,(\vec{b} \times \vec{B})$$

$$|\, F_{QR} \,| = I\, b\, B \sin\theta$$

Forces $\vec{F}_{SP}$ and $\vec{F}_{QR}$ are equal in magnitude but opposite in direction and they cancel out each other. Moreover they act along the same line of action (axis) and hence do not produce torque.

$$\vec{F}_{PQ} = I\,(\vec{l} \times \vec{B})$$

$$|\, F_{PQ} \,| = I\, l\, B \sin 90° = I\, l\, B$$

$$\vec{F}_{RS} = I\,(\vec{l} \times \vec{B})$$

$$|\, F_{Rs} \,| = I\, l\, B \sin 90° = I\, l\, B$$

Forces $\vec{F}_{PQ}$ and $\vec{F}_{RS}$ being equal in magnitude but opposite in direction cancel out each other and do not produce any translational motion. But they act along different lines of action and hence produce torque about the axis of the coil.

Torque experienced by the coil is

$\tau = F_{PQ} \times PN$ (in magnitude)

$\tau = I\,l\,B\,(b\cos\theta)$

$\tau = I\,lb\,B\cos\theta$

$\tau = I\,A\,B\cos\theta$ $(A = lb)$

$\tau = N\,I\,A\,B\cos\theta$ (where N is the no. of turns)

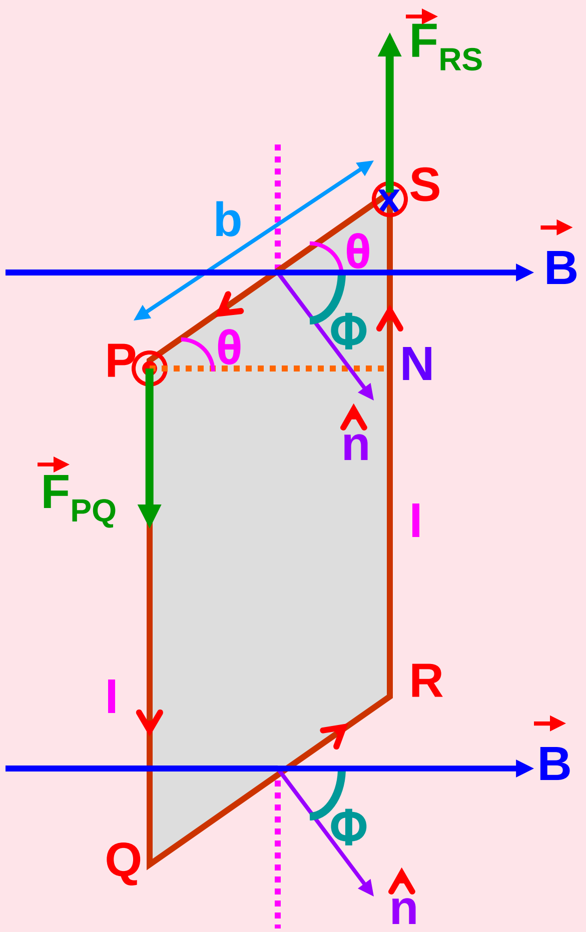
If Φ is the angle between the normal to the coil and the direction of the magnetic field, then

$\Phi + \theta = 90°$ i.e. $\theta = 90° - \Phi$

So,

$\tau = I\,A\,B\cos(90° - \Phi)$

$\tau = N\,I\,A\,B\sin\Phi$



NOTE:

One must be very careful in using the formula in terms of cos or sin since it depends on the angle taken whether with the plane of the coil or the normal of the coil.

Torque in Vector form:

$\tau = N\,I\,A\,B \sin \Phi$

$\vec{\tau} = (N\,I\,A\,B \sin \Phi)\,\hat{n}$ (where $\hat{n}$ is unit vector normal to the plane of the loop)

$\boxed{\vec{\tau} = N\,I\,(\vec{A} \times \vec{B})}$ or $\boxed{\vec{\tau} = N\,(\vec{M} \times \vec{B})}$

(since $\vec{M} = I\,\vec{A}$ is the Magnetic Dipole Moment)

Note:

1) The coil will rotate in the anticlockwise direction (from the top view, according to the figure) about the axis of the coil shown by the dotted line.

2) The torque acts in the upward direction along the dotted line (according to Maxwell's Screw Rule).

3) If $\Phi = 0°$, then $\tau = 0$.

4) If $\Phi = 90°$, then τ is maximum. i.e. $\tau_{max} = N\,I\,A\,B$

5) Units: B in Tesla, I in Ampere, A in m^2 and τ in Nm.

6) The above formulae for torque can be used for any loop irrespective of its shape.

Moving Coil or Suspended Coil or D' Arsonval Type Galvanometer:

Torque experienced by the coil is

$\tau = N\ I\ A\ B \sin \Phi$

Restoring torque in the coil is

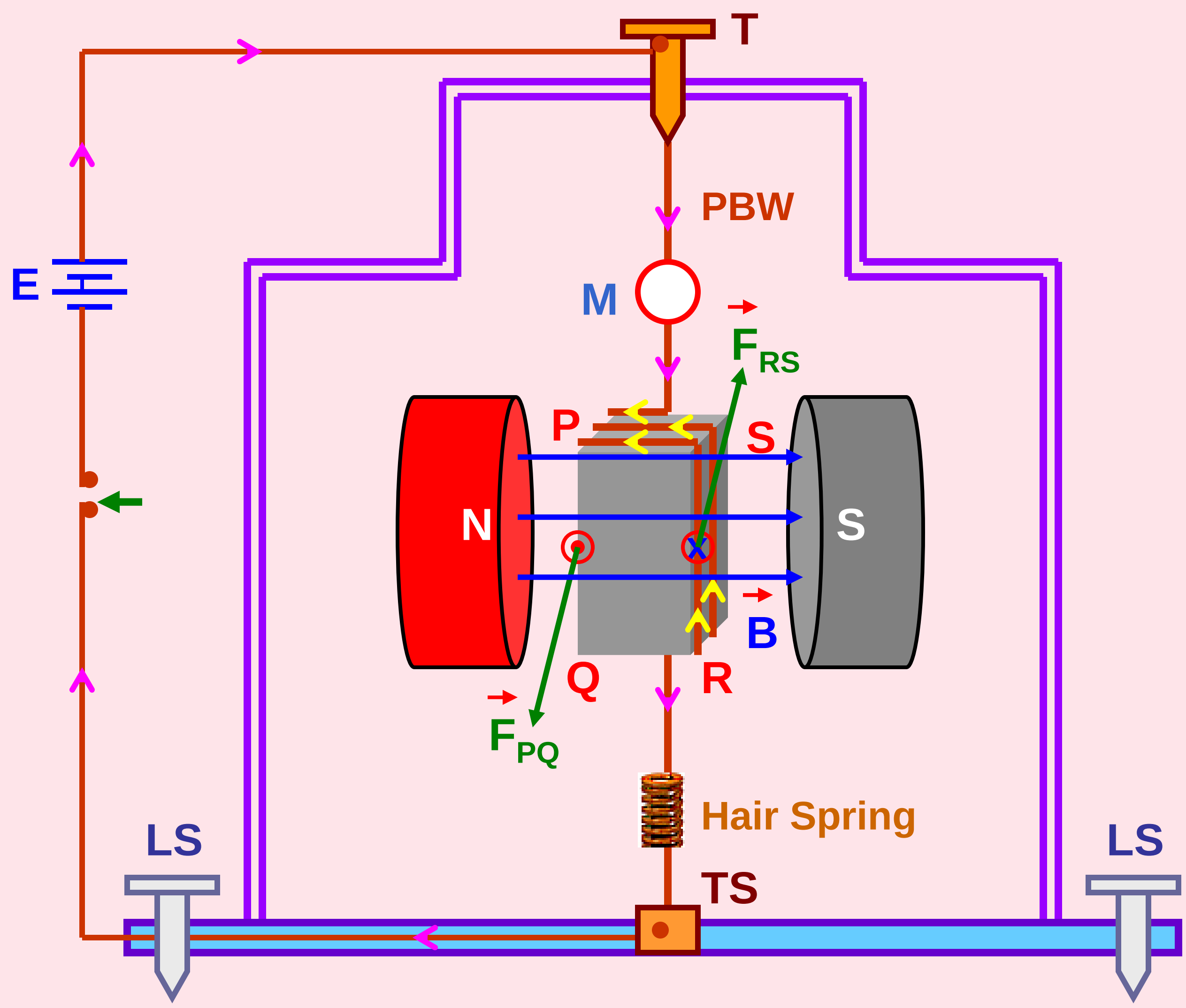
$\tau = k\ \alpha$ (where k is restoring torque per unit angular twist, α is the angular twist in the wire)

At equilibrium,

$N\ I\ A\ B \sin \Phi = k\ \alpha$

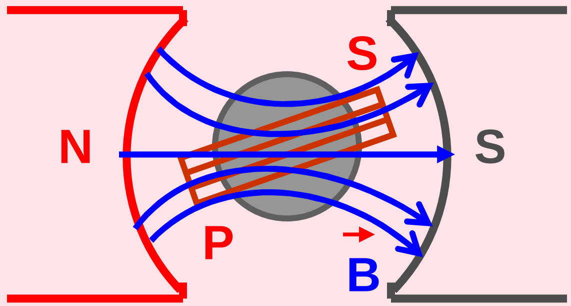
$\therefore\ I = \dfrac{k}{N\ A\ B \sin \Phi}\ \alpha$

The factor sin Φ can be eliminated by choosing Radial Magnetic Field.



T – Torsion Head, TS – Terminal screw, M – Mirror, N,S – Poles pieces of a magnet, LS – Levelling Screws, PQRS – Rectangular coil, PBW – Phosphor Bronze Wire

Radial Magnetic Field:

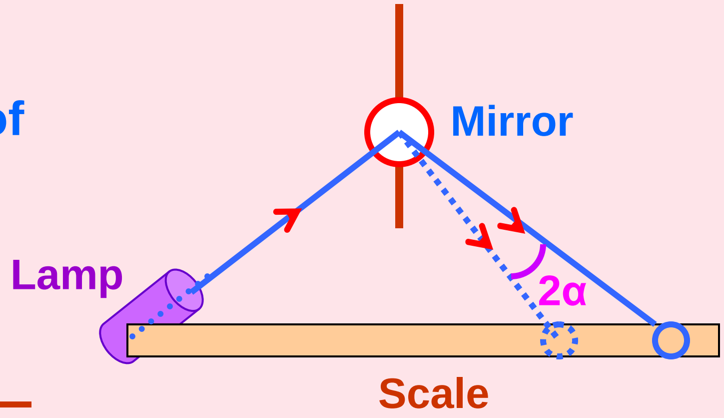


The (top view PS of) plane of the coil PQRS lies along the magnetic lines of force in whichever position the coil comes to rest in equilibrium.

So, the angle between the plane of the coil and the magnetic field is 0°.

or the angle between the normal to the plane of the coil and the magnetic field is 90°.

i.e. $\sin \Phi = \sin 90° = 1$



$\therefore \boxed{I = \dfrac{k}{NAB}\,\alpha}$ or $\boxed{I = G\,\alpha}$ where $G = \dfrac{k}{NAB}$

is called Galvanometer constant

Current Sensitivity of Galvanometer:

It is the defection of galvanometer per unit current.

$$\frac{\alpha}{I} = \frac{NAB}{k}$$

Voltage Sensitivity of Galvanometer:

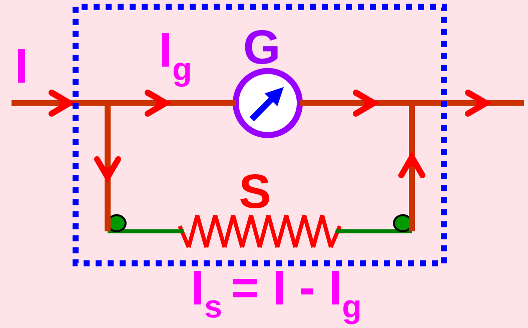
It is the defection of galvanometer per unit voltage.

$$\frac{\alpha}{V} = \frac{NAB}{kR}$$

Conversion of Galvanometer to Ammeter:

Galvanometer can be converted into ammeter by shunting it with a very small resistance.

Potential difference across the galvanometer and shunt resistance are equal.

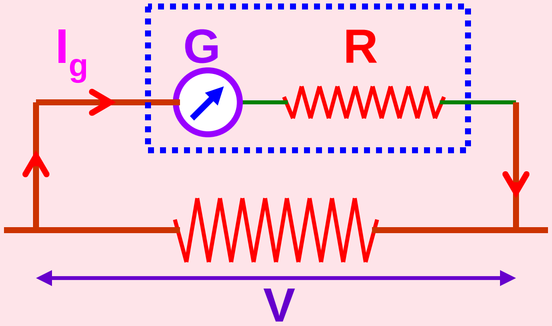


$\therefore$ $(I - I_g)\,S = I_g\,G$ or $S = \dfrac{I_g\,G}{I - I_g}$

Conversion of Galvanometer to Voltmeter:

Galvanometer can be converted into voltmeter by connecting it with a very high resistance.

Potential difference across the given load resistance is the sum of p.d across galvanometer and p.d. across the high resistance.



$\therefore$ $V = I_g\,(G + R)$ or $R = \dfrac{V}{I_g} - G$

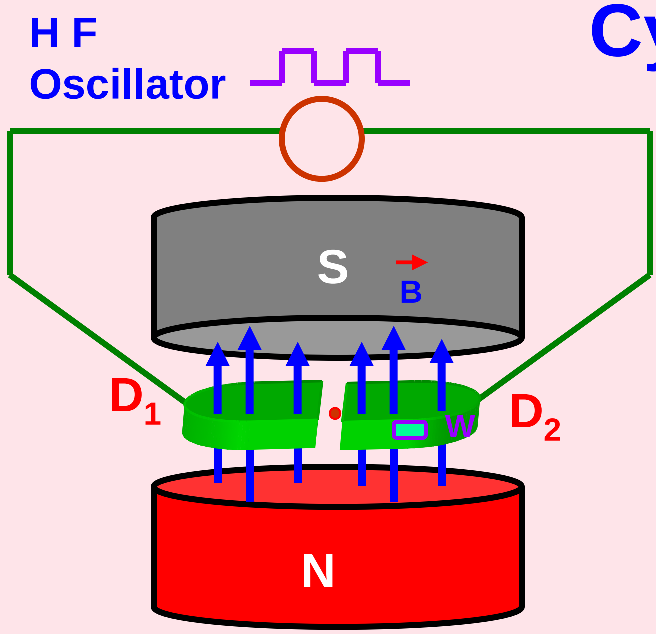
Difference between Ammeter and Voltmeter:

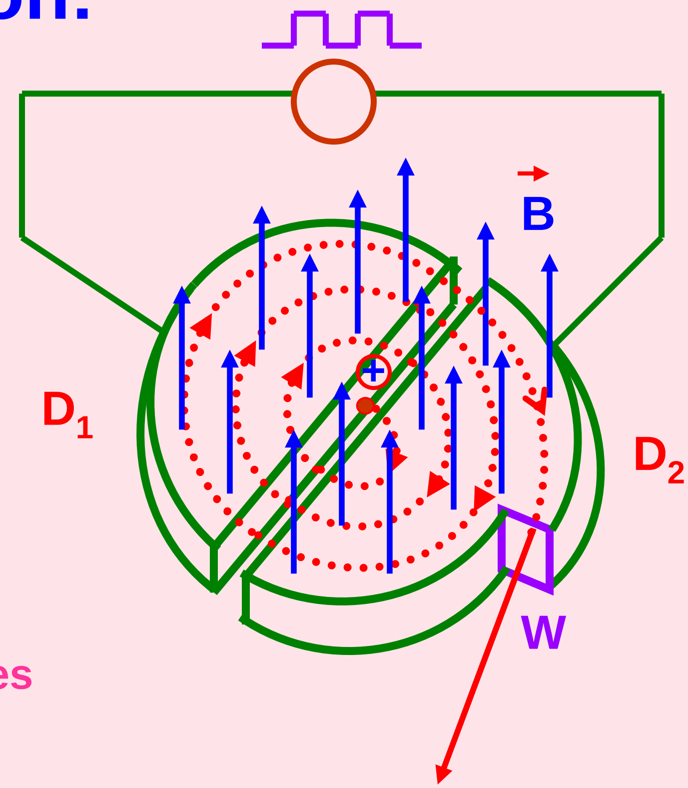
S.No.	Ammeter	Voltmeter
1	It is a low resistance instrument.	It is a high resistance instrument.
2	Resistance is GS / (G + S)	Resistance is G + R
3	Shunt Resistance is $(GI_g) / (I - I_g)$ and is very small.	Series Resistance is $(V / I_g) - G$ and is very high.
4	It is always connected in series.	It is always connected in parallel.
5	Resistance of an ideal ammeter is zero.	Resistance of an ideal voltmeter is infinity.
6	Its resistance is less than that of the galvanometer.	Its resistance is greater than that of the voltmeter.
7	It is not possible to decrease the range of the given ammeter.	It is possible to decrease the range of the given voltmeter.

MAGNETIC EFFECT OF CURRENT - III

1. Cyclotron
2. Ampere’s Circuital Law
3. Magnetic Field due to a Straight Solenoid
4. Magnetic Field due to a Toroidal Solenoid

Cyclotron:





D_1, D_2 – Dees N, S – Magnetic Pole Pieces

W – Window B - Magnetic Field

Working: Imagining D_1 is positive and D_2 is negative, the + vely charged particle kept at the centre and in the gap between the dees get accelerated towards D_2. Due to perpendicular magnetic field and according to Fleming's Left Hand Rule the charge gets deflected and describes semi-circular path.

When it is about to leave D_2, D_2 becomes + ve and D_1 becomes – ve. Therefore the particle is again accelerated into D_1 where it continues to describe the semi-circular path. The process continues till the charge traverses through the whole space in the dees and finally it comes out with very high speed through the window.

Theory:

The magnetic force experienced by the charge provides centripetal force required to describe circular path.

$\therefore \quad mv^2 / r = qvB \sin 90°$

$$v = \frac{B\,q\,r}{m}$$

(where m – mass of the charged particle, q – charge, v – velocity on the path of radius – r, B is magnetic field and 90° is the angle b/n v and B)

If t is the time taken by the charge to describe the semi-circular path inside the dee, then

$$t = \frac{\pi\ r}{v} \quad \text{or} \quad t = \frac{\pi\ m}{B\,q}$$

Time taken inside the dee depends only on the magnetic field and m/q ratio and not on the speed of the charge or the radius of the path.

If T is the time period of the high frequency oscillator, then for resonance,

$$T = 2\,t \quad \text{or} \quad T = \frac{2\pi m}{B\,q}$$

If f is the frequency of the high frequency oscillator (Cyclotron Frequency), then

$$\boxed{f = \frac{B\,q}{2\pi m}}$$

Maximum Energy of the Particle:

Kinetic Energy of the charged particle is

$$K.E. = \tfrac{1}{2}\, m\, v^2 = \tfrac{1}{2}\, m \left(\frac{B\, q\, r}{m}\right)^2 = \tfrac{1}{2}\, \frac{B^2\, q^2\, r^2}{m}$$

Maximum Kinetic Energy of the charged particle is when $r = R$ (radius of the D's).

$$\boxed{K.E._{max} = \tfrac{1}{2}\, \frac{B^2\, q^2\, R^2}{m}}$$

The expressions for Time period and Cyclotron frequency only when m remains constant. (Other quantities are already constant.)

But m varies with v according to Einstein's Relativistic Principle as per

$$m = \frac{m_0}{[1 - (v^2 / c^2)]^{1/2}}$$

If frequency is varied in synchronisation with the variation of mass of the charged particle (by maintaining B as constant) to have resonance, then the cyclotron is called synchro – cyclotron.

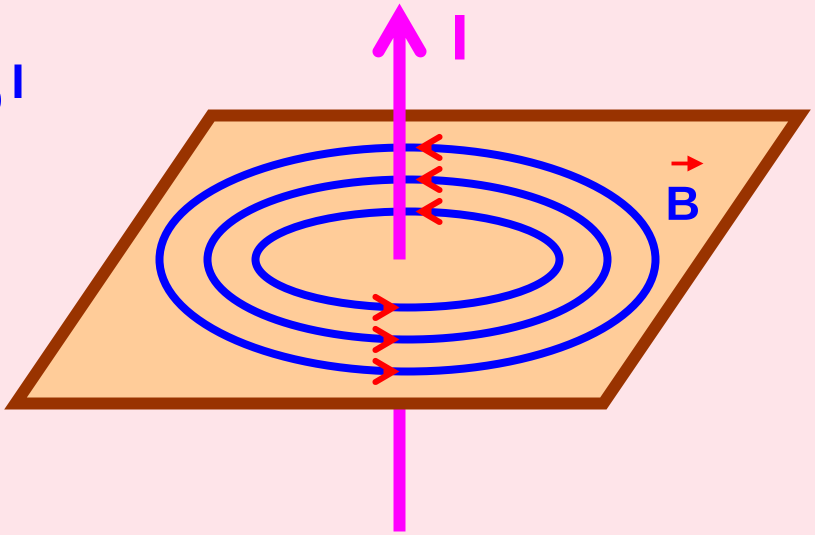
If magnetic field is varied in synchronisation with the variation of mass of the charged particle (by maintaining f as constant) to have resonance, then the cyclotron is called isochronous – cyclotron.

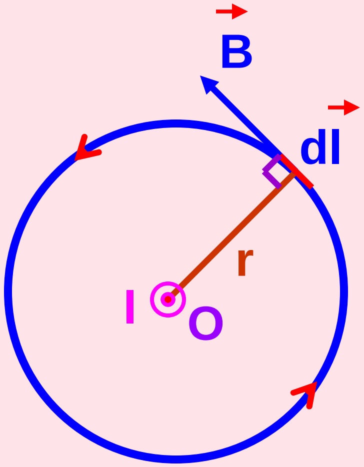
NOTE: Cyclotron can not be used for accelerating neutral particles. Electrons can not be accelerated because they gain speed very quickly due to their lighter mass and go out of phase with alternating e.m.f. and get lost within the dees.

Ampere's Circuital Law:

The line integral $\oint \vec{B} \cdot \vec{dl}$ for a closed curve is equal to μ_0 times the net current I threading through the area bounded by the curve.

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$





Current is emerging out and the magnetic field is anticlockwise.

Proof:

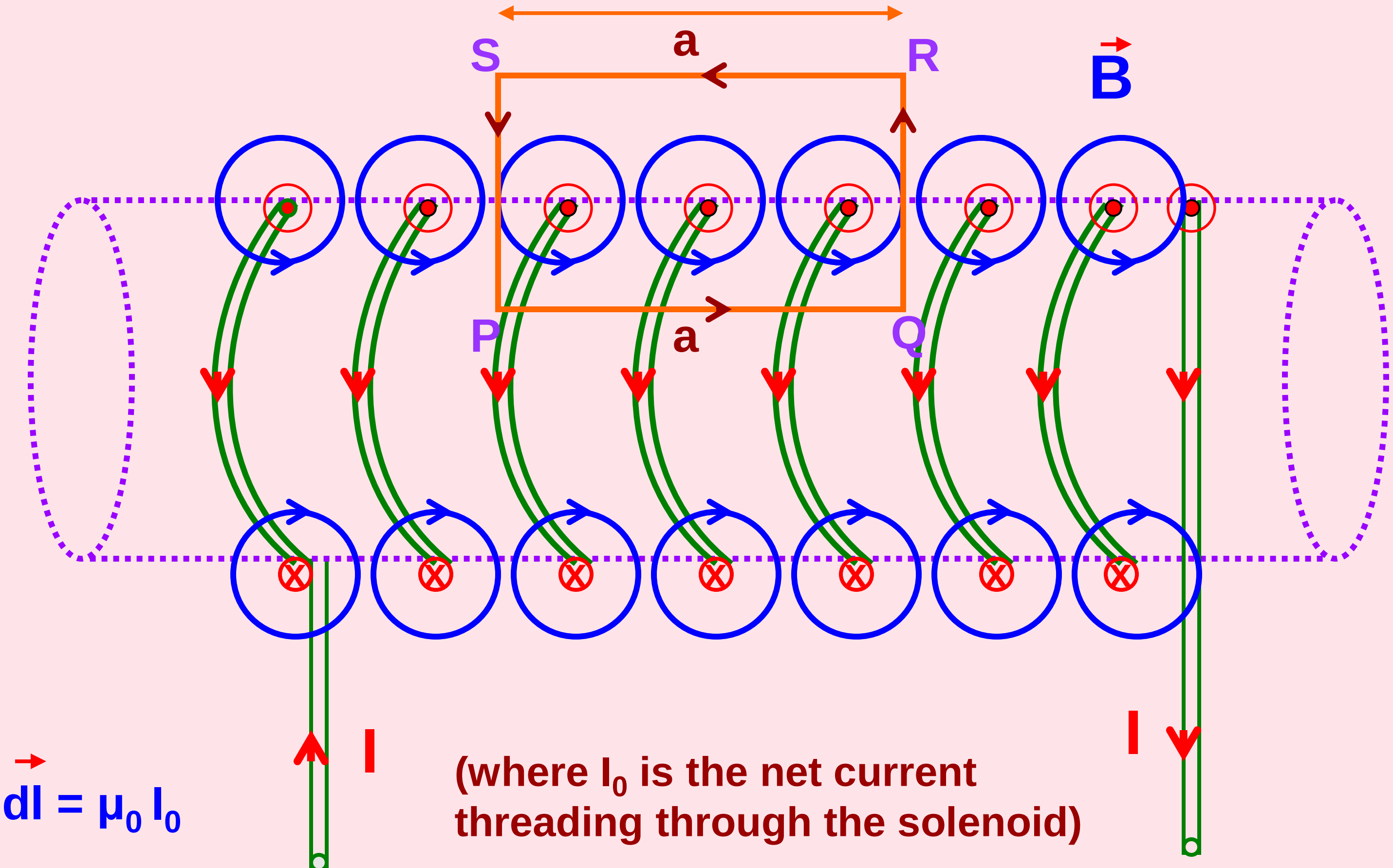
$$\oint \vec{B} \cdot \vec{dl} = \oint B \cdot dl \cos 0°$$

$$= \oint B \cdot dl = B \oint dl$$

$$= B\,(2\pi\, r) \quad = (\mu_0\, I / 2\pi\, r) \times 2\pi\, r$$

$$\boxed{\oint \vec{B} \cdot \vec{dl} = \mu_0 I}$$

Magnetic Field at the centre of a Straight Solenoid:



$\oint \vec{B} \,.\, \vec{dl} = \mu_0 \, I_0$

(where I_0 is the net current threading through the solenoid)

$$\oint \vec{B} \,.\, \vec{dl} = \oint_{PQ} \vec{B} \,.\, \vec{dl} + \oint_{QR} \vec{B} \,.\, \vec{dl} + \oint_{RS} \vec{B} \,.\, \vec{dl} + \oint_{SP} \vec{B} \,.\, \vec{dl}$$

$$= \oint B \,.\, dl \cos 0° + \oint B \,.\, dl \cos 90° + \oint 0 \,.\, dl \cos 0° + \oint B \,.\, dl \cos 90°$$

$= B \oint dl = B.a$ and $\mu_0 \, I_0 = \mu_0 \, n \, a \, I$ $\therefore$ $\boxed{B = \mu_0 \, n \, I}$

(where n is no. of turns per unit length, a is the length of the path and I is the current passing through the lead of the solenoid)

Magnetic Field due to Toroidal Solenoid (Toroid):

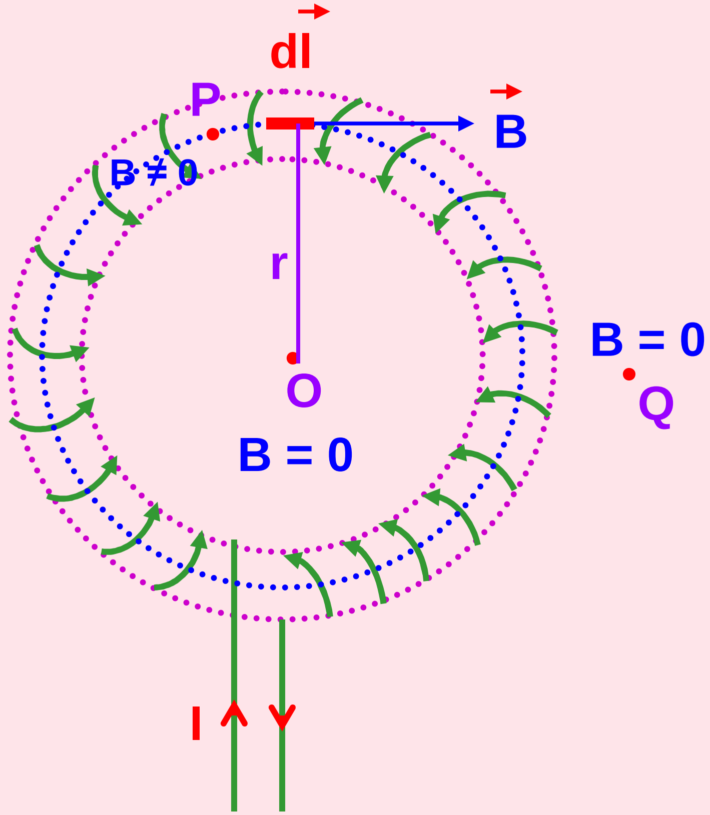
$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_0$$

$$\oint \vec{B} \cdot \vec{dl} = \oint B \cdot dl \cos 0°$$

$$= B \oint dl = B (2\pi r)$$

And $\mu_0 I_0 = \mu_0 n (2\pi r) I$

$$\therefore \boxed{B = \mu_0 n I}$$



NOTE:

The magnetic field exists only in the tubular area bound by the coil and it does not exist in the area inside and outside the toroid.

i.e. B is zero at O and Q and non-zero at P.

MAGNETISM

1. Bar Magnet and its properties
2. Current Loop as a Magnetic Dipole and Dipole Moment
3. Current Solenoid equivalent to Bar Magnet
4. Bar Magnet and it Dipole Moment
5. Coulomb's Law in Magnetism
6. Important Terms in Magnetism
7. Magnetic Field due to a Magnetic Dipole
8. Torque and Work Done on a Magnetic Dipole
9. Terrestrial Magnetism
10. Elements of Earth's Magnetic Field
11. Tangent Law
12. Properties of Dia-, Para- and Ferro-magnetic substances
13. Curie's Law in Magnetism
14. Hysteresis in Magnetism

Magnetism:

- Phenomenon of attracting magnetic substances like iron, nickel, cobalt, etc.

- **A body possessing the property of magnetism is called a magnet.**
- **A magnetic pole is a point near the end of the magnet where magnetism is concentrated.**
- **Earth is a natural magnet.**
- **The region around a magnet in which it exerts forces on other magnets and on objects made of iron is a magnetic field.**

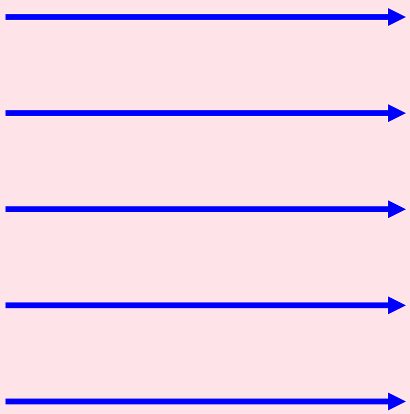
Properties of a bar magnet:

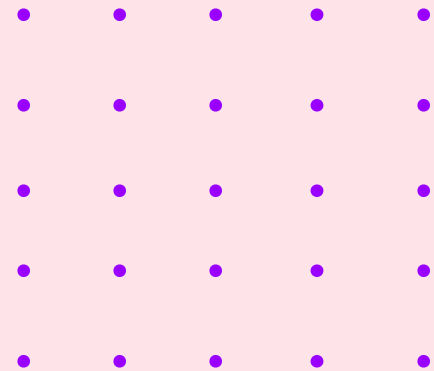
1. **A freely suspended magnet aligns itself along North – South direction.**
2. **Unlike poles attract and like poles repel each other.**
3. **Magnetic poles always exist in pairs. i.e. Poles can not be separated.**
4. **A magnet can induce magnetism in other magnetic substances.**
5. **It attracts magnetic substances.**

Repulsion is the surest test of magnetisation: **A magnet attracts iron rod as well as opposite pole of other magnet. Therefore it is not a sure test of magnetisation.**

But, if a rod is repelled with strong force by a magnet, then the rod is surely magnetised.

Representation of Uniform Magnetic Field:



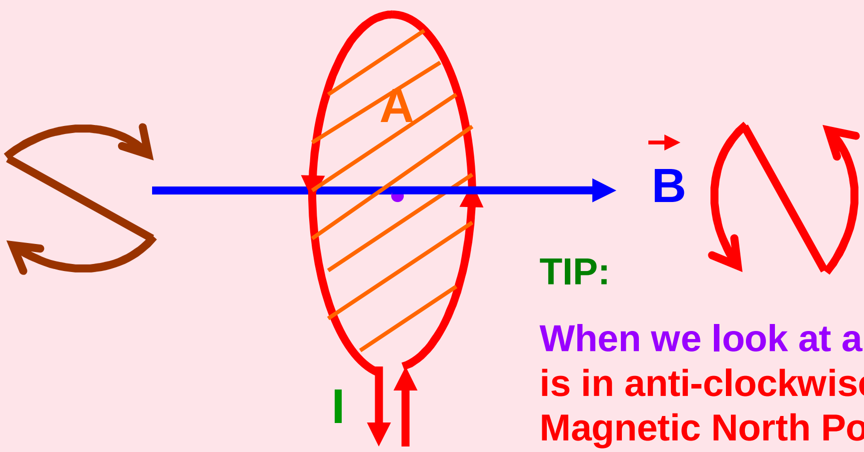


Uniform field on the plane of the diagram

Uniform field perpendicular & into the plane of the diagram

Uniform field perpendicular & emerging out of the plane of the diagram

Current Loop as a Magnetic Dipole & Dipole Moment:



Magnetic Dipole Moment is

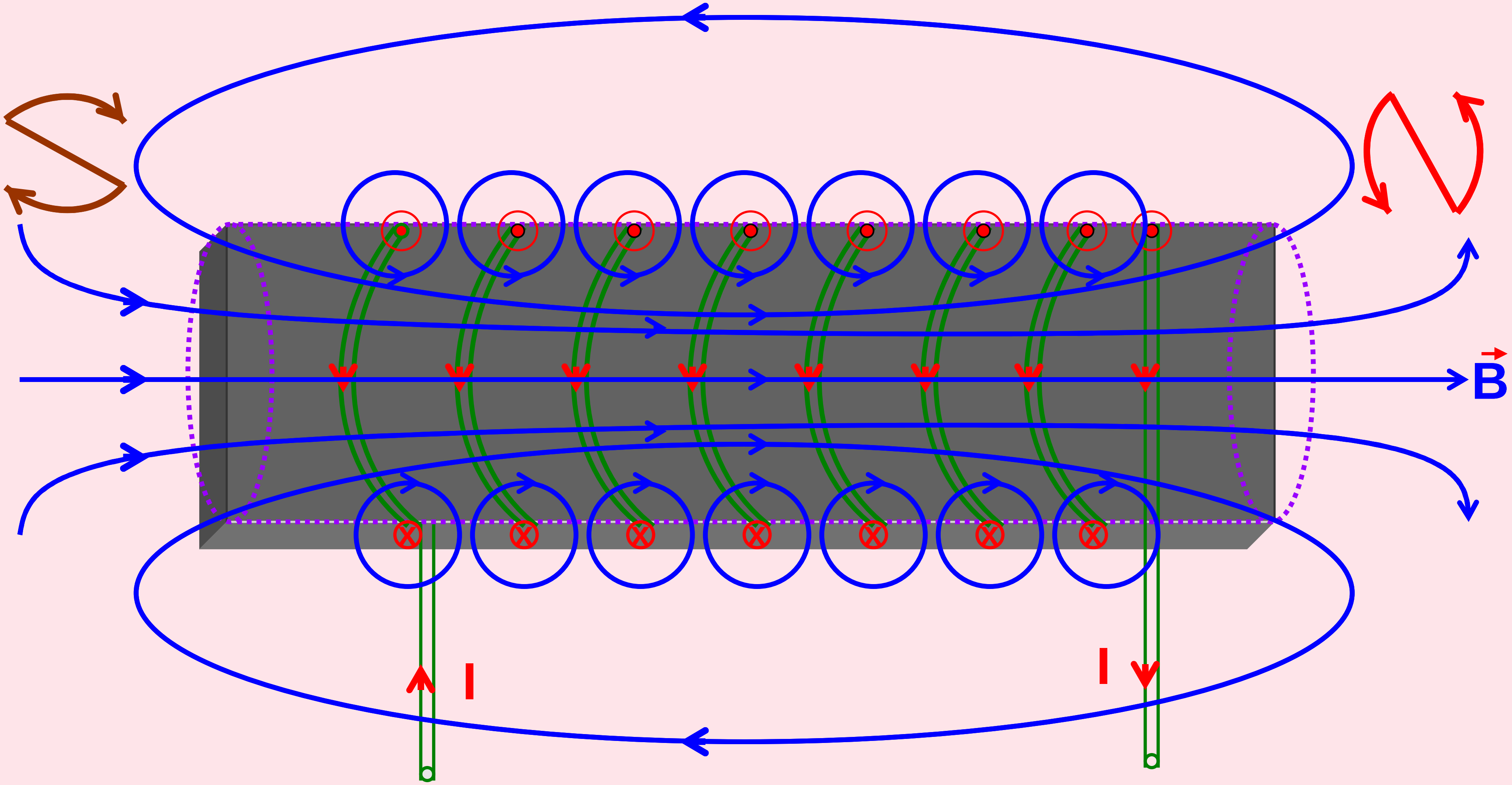
$$\boxed{\vec{M} = I\,A\,\hat{n}}$$

SI unit is A m^2.

TIP:

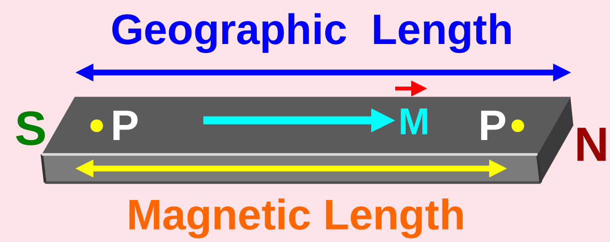
When we look at any one side of the loop carrying current, if the current is in anti-clockwise direction then that side of the loop behaves like Magnetic North Pole and if the current is in clockwise direction then that side of the loop behaves like Magnetic South Pole.

Current Solenoid as a Magnetic Dipole or Bar Magnet:



TIP: Play previous and next to understand the similarity of field lines.

Bar Magnet:



1. The line joining the poles of the magnet is called magnetic axis.
2. The distance between the poles of the magnet is called magnetic length of the magnet.
3. The distance between the ends of the magnet is called the geometrical length of the magnet.
4. The ratio of magnetic length and geometrical length is nearly 0.84.

Magnetic Dipole & Dipole Moment:

A pair of magnetic poles of equal and opposite strengths separated by a finite distance is called a magnetic dipole.

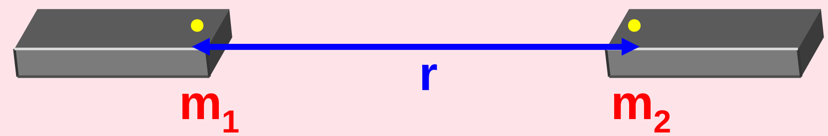
The magnitude of dipole moment is the product of the pole strength m and the separation 2l between the poles.

Magnetic Dipole Moment is $\boxed{\vec{M} = m.2l.\hat{l}}$ SI unit of pole strength is A.m

The direction of the dipole moment is from South pole to North Pole along the axis of the magnet.

Coulomb's Law in Magnetism:

The force of attraction or repulsion between two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of the distance between them.



$F \ \alpha \ m_1 \ m_2$

$\alpha \ r^2$

$$F = \frac{k \ m_1 \ m_2}{r^2} \quad \text{or} \quad \boxed{F = \frac{\mu_0 \ m_1 \ m_2}{4\pi \quad r^2}}$$

(where $k = \mu_0 / 4\pi$ is a constant and $\mu_0 = 4\pi \times 10^{-7}$ T m A-1)

In vector form

$$\vec{F} = \frac{\mu_0 \ m_1 \ m_2 \ \hat{r}}{4\pi \quad r^2}$$

$$\boxed{\vec{F} = \frac{\mu_0 \ m_1 \ m_2 \ \vec{r}}{4\pi \quad r^3}}$$

Magnetic Intensity or Magnetising force (H):

i) Magnetic Intensity at a point is the force experienced by a north pole of unit pole strength placed at that point due to pole strength of the given magnet. $H = B / \mu$

ii) It is also defined as the magnetomotive force per unit length.

iii) It can also be defined as the degree or extent to which a magnetic field can magnetise a substance.

iv) It can also be defined as the force experienced by a unit positive charge flowing with unit velocity in a direction normal to the magnetic field.

v) Its SI unit is ampere-turns per linear metre.

vi) Its cgs unit is oersted.

Magnetic Field Strength or Magnetic Field or Magnetic Induction or Magnetic Flux Density (B):

i) Magnetic Flux Density is the number of magnetic lines of force passing normally through a unit area of a substance. $B = \mu H$

ii) Its SI unit is weber-m^{-2} or Tesla (T).

iii) Its cgs unit is gauss. 1 gauss = 10^{-4} Tesla

Magnetic Flux (Φ):

i) It is defined as the number of magnetic lines of force passing normally through a surface.

ii) Its SI unit is weber.

Relation between B and H:

$B = \mu H$ (where μ is the permeability of the medium)

Magnetic Permeability (μ):

It is the degree or extent to which magnetic lines of force can pass enter a substance.

Its SI unit is $T\ m\ A^{-1}$ or $wb\ A^{-1}\ m^{-1}$ or $H\ m^{-1}$

Relative Magnetic Permeability (μ_r):

It is the ratio of magnetic flux density in a material to that in vacuum.

It can also be defined as the ratio of absolute permeability of the material to that in vacuum.

$\mu_r = B / B_0$ or $\mu_r = \mu / \mu_0$

Intensity of Magnetisation: (I):

i) It is the degree to which a substance is magnetised when placed in a magnetic field.

ii) It can also be defined as the magnetic dipole moment (M) acquired per unit volume of the substance (V).

iii) It can also be defined as the pole strength (m) per unit cross-sectional area (A) of the substance.

iv) $I = M / V$

v) $I = m(2l) / A(2l) = m / A$

vi) SI unit of Intensity of Magnetisation is $A\ m^{-1}$.

Magnetic Susceptibility (c_m):

i) It is the property of the substance which shows how easily a substance can be magnetised.

ii) It can also be defined as the ratio of intensity of magnetisation (I) in a substance to the magnetic intensity (H) applied to the substance.

iii) $c_m = I / H$

Susceptibility has no unit.

Relation between Magnetic Permeability (μ_r) & Susceptibility (c_m):

$$\mu_r = 1 + c_m$$

Magnetic Field due to a Magnetic Dipole (Bar Magnet):

i) At a point on the axial line of the magnet:

$$B_P = \frac{\mu_0 \; 2 M x}{4\pi \; (x^2 - l^2)^2}$$

If l << x, then

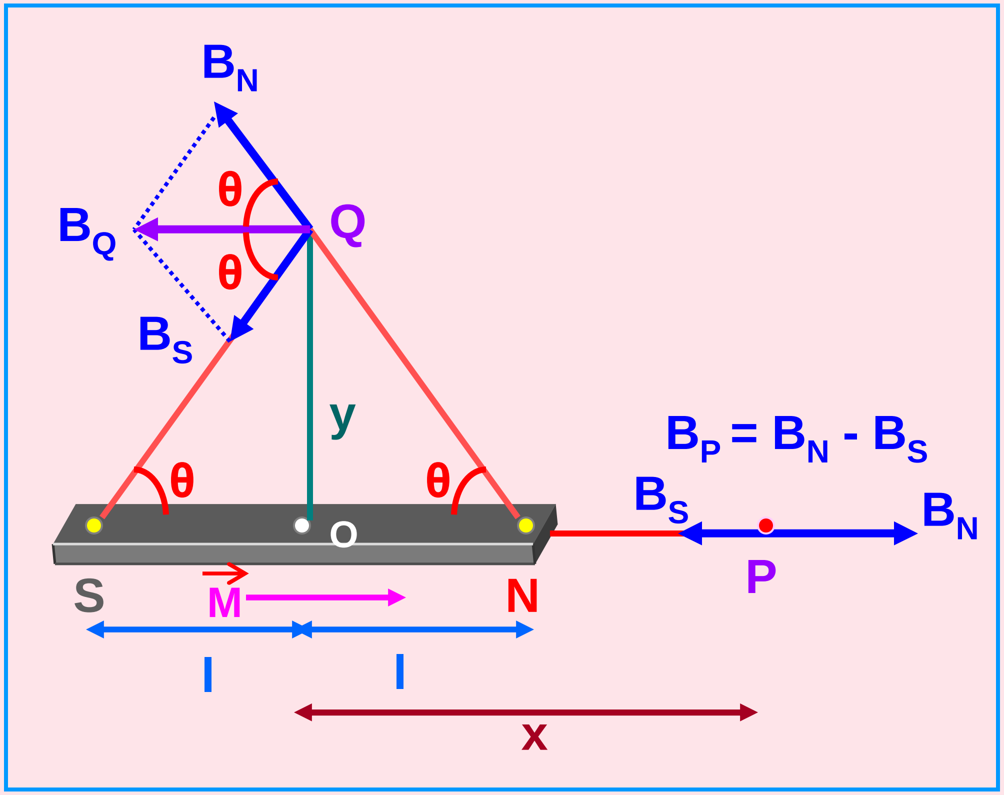
$$B_P \approx \frac{\mu_0 \; 2 M}{4\pi \; x^3}$$

ii) At a point on the equatorial line of the magnet:

$$B_Q = \frac{\mu_0 \; M}{4\pi \; (y^2 + l^2)^{3/2}}$$

If l << y, then

$$B_P \approx \frac{\mu_0 \; M}{4\pi \; y^3}$$

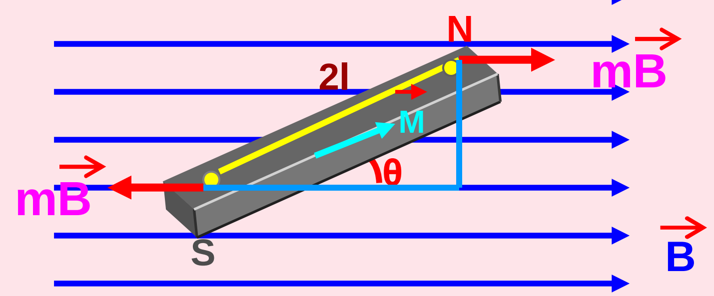


Magnetic Field at a point on the axial line acts along the dipole moment vector.

Magnetic Field at a point on the equatorial line acts opposite to the dipole moment vector.

Torque on a Magnetic Dipole (Bar Magnet) in Uniform Magnetic Field:

The forces of magnitude mB act opposite to each other and hence net force acting on the bar magnet due to external uniform magnetic field is zero. So, there is no translational motion of the magnet.



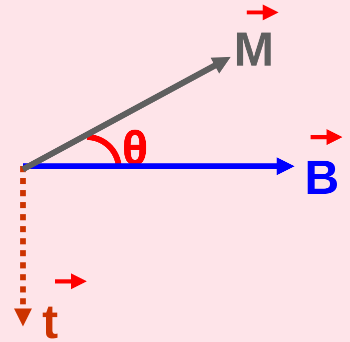
However the forces are along different lines of action and constitute a couple. Hence the magnet will rotate and experience torque.

Torque = Magnetic Force x ⊥ distance

$$t = mB\,(2l \sin\theta)$$

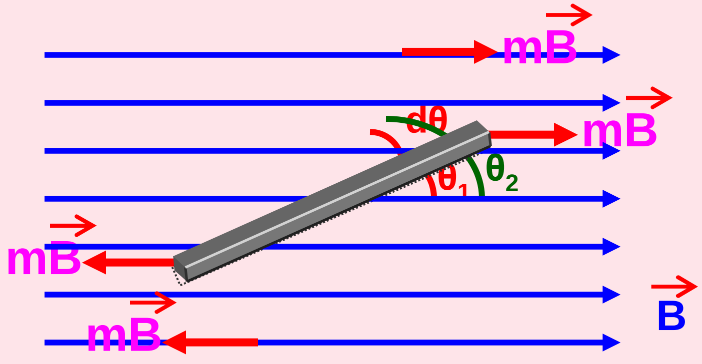
$$= M B \sin\theta$$

$$\boxed{\vec{t} = \vec{M} \times \vec{B}}$$



Direction of Torque is perpendicular and into the plane containing $\vec{M}$ and $\vec{B}$.

Work done on a Magnetic Dipole (Bar Magnet) in Uniform Magnetic Field:



$dW = \tau d\theta$

$= M\,B \sin\theta\, d\theta$

$$W = \int_{\theta_1}^{\theta_2} M\,B \sin\theta\, d\theta$$

$$W = M\,B\,(\cos\theta_1 - \cos\theta_2)$$

If Potential Energy is arbitrarily taken zero when the dipole is at 90°, then P.E in rotating the dipole and inclining it at an angle θ is

Potential Energy = $- M\,B \cos\theta$

Note:

Potential Energy can be taken zero arbitrarily at any position of the dipole.

Terrestrial Magnetism:

i) <u>Geographic Axis</u> is a straight line passing through the geographical poles of the earth. It is the axis of rotation of the earth. It is also known as polar axis.

ii) <u>Geographic Meridian</u> at any place is a vertical plane passing through the geographic north and south poles of the earth.

iii) <u>Geographic Equator</u> is a great circle on the surface of the earth, in a plane perpendicular to the geographic axis. All the points on the geographic equator are at equal distances from the geographic poles.

iv) <u>Magnetic Axis</u> is a straight line passing through the magnetic poles of the earth. It is inclined to Geographic Axis nearly at an angle of 17°.

v) <u>Magnetic Meridian</u> at any place is a vertical plane passing through the magnetic north and south poles of the earth.

vi) <u>Magnetic Equator</u> is a great circle on the surface of the earth, in a plane perpendicular to the magnetic axis. All the points on the magnetic equator are at equal distances from the magnetic poles.

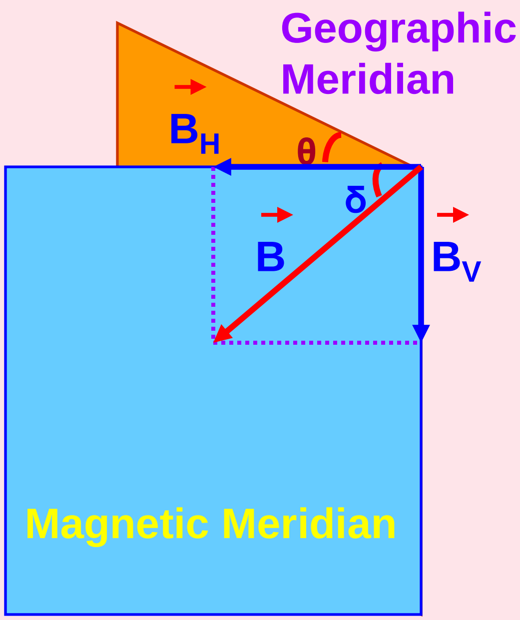
Declination (θ):

The angle between the magnetic meridian and the geographic meridian at a place is Declination at that place.

It varies from place to place.

Lines shown on the map through the places that have the same declination are called isogonic line.

Line drawn through places that have zero declination is called an agonic line.



Dip or Inclination (δ):

The angle between the horizontal component of earth's magnetic field and the earth's resultant magnetic field at a place is Dip or Inclination at that place.

It is zero at the equator and 90° at the poles.

Lines drawn up on a map through places that have the same dip are called isoclinic lines.

The line drawn through places that have zero dip is known as an aclinic line. It is the magnetic equator.

Horizontal Component of Earth's Magnetic Field (B_H):

The total intensity of the earth's magnetic field does not lie in any horizontal plane. Instead, it lies along the direction at an angle of dip (δ) to the horizontal. The component of the earth's magnetic field along the horizontal at an angle δ is called Horizontal Component of Earth's Magnetic Field.

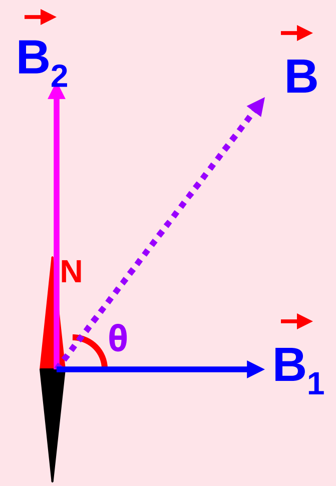
$$B_H = B \cos \delta$$

Similarly Vertical Component is $B_V = B \sin \delta$

such that $B = \sqrt{B_H^2 + B_V^2}$

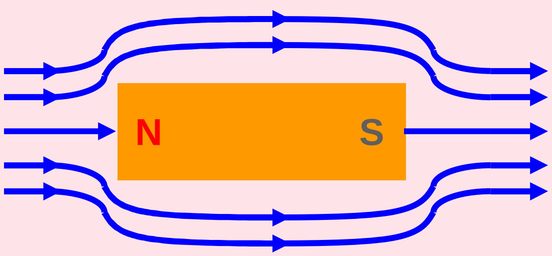
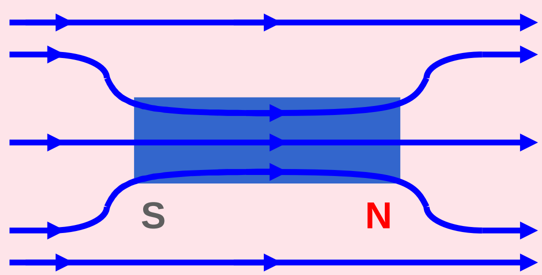
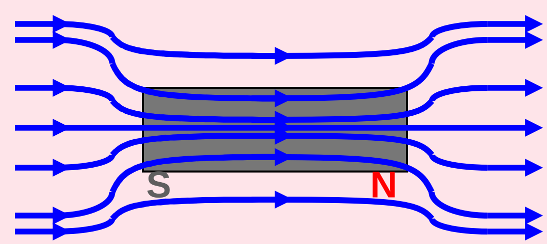
Tangent Law:

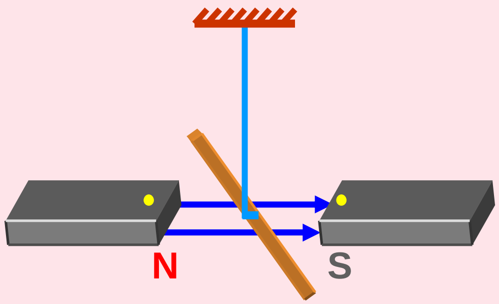
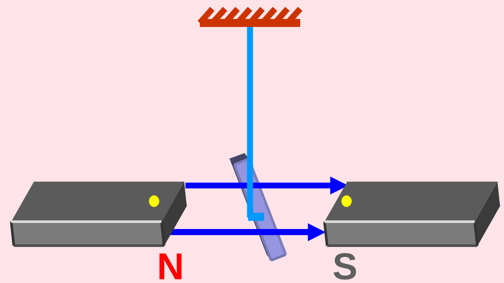
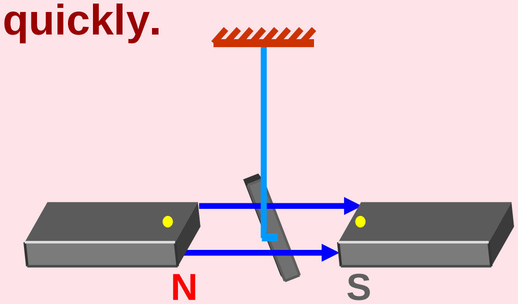
If a magnetic needle is suspended in a region where two uniform magnetic fields are perpendicular to each other, the needle will align itself along the direction of the resultant field of the two fields at an angle θ such that the tangent of the angle is the ratio of the two fields.

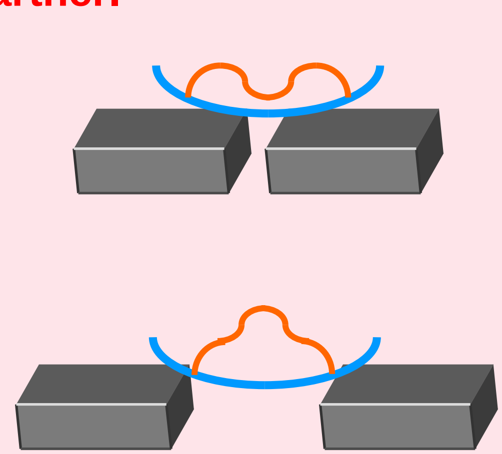
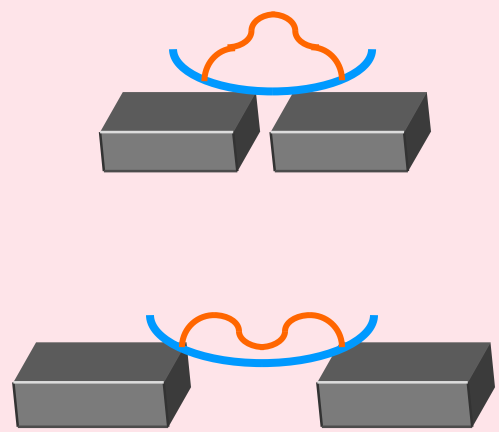
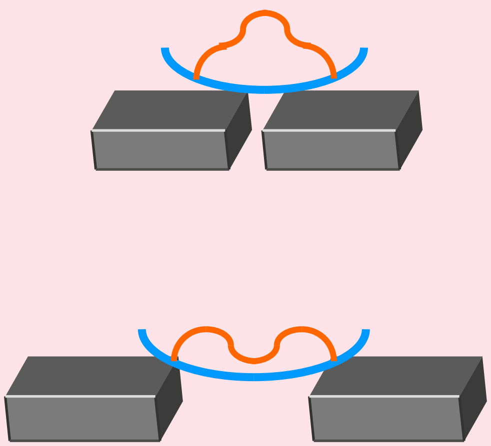


$$\boxed{\tan \theta = B_2 / B_1}$$

Comparison of Dia, Para and Ferro Magnetic materials:

DIA	PARA	FERRO
1. Diamagnetic substances are those substances which are feebly repelled by a magnet. Eg. Antimony, Bismuth, Copper, Gold, Silver, Quartz, Mercury, Alcohol, water, Hydrogen, Air, Argon, etc.	Paramagnetic substances are those substances which are feebly attracted by a magnet. Eg. Aluminium, Chromium, Alkali and Alkaline earth metals, Platinum, Oxygen, etc.	Ferromagnetic substances are those substances which are strongly attracted by a magnet. Eg. Iron, Cobalt, Nickel, Gadolinium, Dysprosium, etc.
2. When placed in magnetic field, the lines of force tend to avoid the substance. 	The lines of force prefer to pass through the substance rather than air. 	The lines of force tend to crowd into the specimen. 

2. When placed in non-uniform magnetic field, it moves from stronger to weaker field (feeble repulsion).	**When placed in non-uniform magnetic field, it moves from weaker to stronger field (feeble attraction).**	**When placed in non-uniform magnetic field, it moves from weaker to stronger field (strong attraction).**
3. When a diamagnetic rod is freely suspended in a uniform magnetic field, it aligns itself in a direction perpendicular to the field. 	**When a paramagnetic rod is freely suspended in a uniform magnetic field, it aligns itself in a direction parallel to the field.** 	**When a paramagnetic rod is freely suspended in a uniform magnetic field, it aligns itself in a direction parallel to the field very quickly.** 

4. If diamagnetic liquid taken in a watch glass is placed in uniform magnetic field, it collects away from the centre when the magnetic poles are closer and collects at the centre when the magnetic poles are farther.	If paramagnetic liquid taken in a watch glass is placed in uniform magnetic field, it collects at the centre when the magnetic poles are closer and collects away from the centre when the magnetic poles are farther.	If ferromagnetic liquid taken in a watch glass is placed in uniform magnetic field, it collects at the centre when the magnetic poles are closer and collects away from the centre when the magnetic poles are farther.
		

5. When a diamagnetic substance is placed in a magnetic field, it is weakly magnetised in the direction opposite to the inducing field.	When a paramagnetic substance is placed in a magnetic field, it is weakly magnetised in the direction of the inducing field.	When a ferromagnetic substance is placed in a magnetic field, it is strongly magnetised in the direction of the inducing field.
6. Induced Dipole Moment (M) is a small – ve value.	Induced Dipole Moment (M) is a small + ve value.	Induced Dipole Moment (M) is a large + ve value.
7. Intensity of Magnetisation (I) has a small – ve value.	Intensity of Magnetisation (I) has a small + ve value.	Intensity of Magnetisation (I) has a large + ve value.
8. Magnetic permeability μ is always less than unity.	Magnetic permeability μ is more than unity.	Magnetic permeability μ is large i.e. much more than unity.

9. Magnetic susceptibility c_m has a small – ve value.	Magnetic susceptibility c_m has a small + ve value.	Magnetic susceptibility c_m has a large + ve value.
10. They do not obey Curie's Law. i.e. their properties do not change with temperature.	They obey Curie's Law. They lose their magnetic properties with rise in temperature.	They obey Curie's Law. At a certain temperature called Curie Point, they lose ferromagnetic properties and behave like paramagnetic substances.

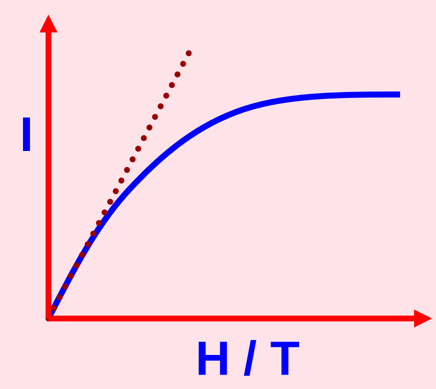
Curie's Law:

Magnetic susceptibility of a material varies inversely with the absolute temperature.

$I \propto H / T$ or $I / H \propto 1 / T$

$c_m \propto 1 / T$

$c_m = C / T$ (where C is Curie constant)



Curie temperature for iron is 1000 K, for cobalt 1400 K and for nickel 600 K.

Hysteresis Loop or Magnetisation Curve:

Intensity of Magnetisation (I) increases with increase in Magnetising Force (H) initially through OA and reaches saturation at A.

When H is decreased, I decreases but it does not come to zero at H = 0.

The residual magnetism (I) set up in the material represented by OB is called Retentivity.

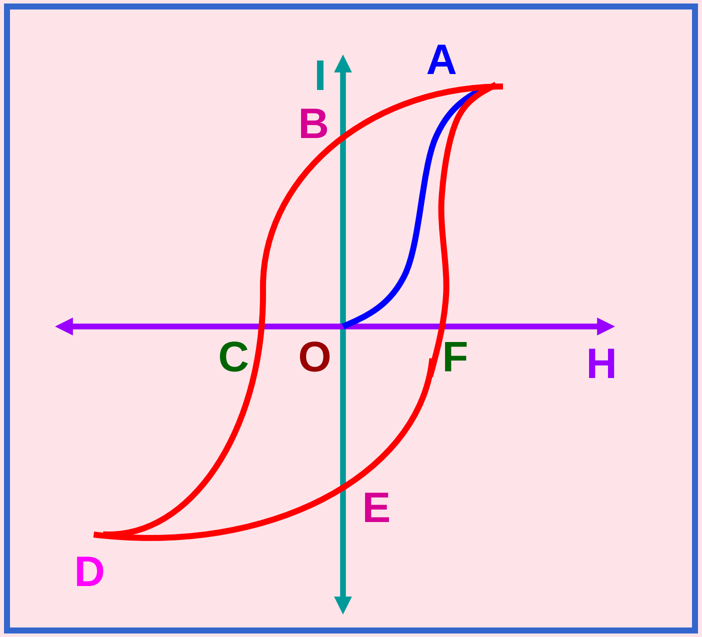
To bring I to zero (to demagnetise completely), opposite (negative) magnetising force is applied. This magetising force represented by OC is called coercivity.

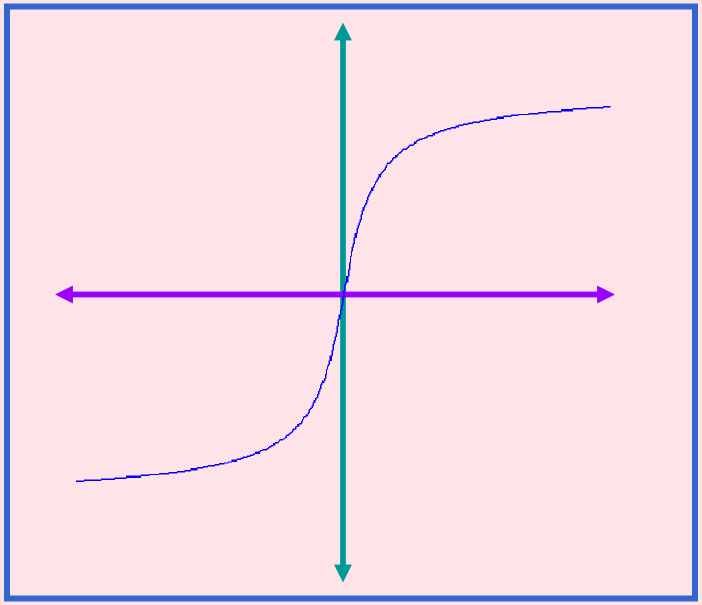
After reaching the saturation level D, when the magnetising force is reversed, the curve closes to the point A completing a cycle.

The loop ABCDEFA is called Hysteresis Loop.

The area of the loop gives the loss of energy due to the cycle of magnetisation and demagnetisation and is dissipated in the form of heat.

The material (like iron) having thin loop is used for making temporary magnets and that with thick loop (like steel) is used for permanent magnets.





Question 4.1:

A circular coil of wire consisting of 100 turns, each of radius 8.0 cm carries a current of 0.40 A. What is the magnitude of the magnetic field **B** at the centre of the coil?

Answer

Number of turns on the circular coil, $n = 100$

Radius of each turn, $r = 8.0$ cm $= 0.08$ m

Current flowing in the coil, $I = 0.4$ A

Magnitude of the magnetic field at the centre of the coil is given by the relation,

$$|\mathbf{B}| = \frac{\mu_0}{4\pi}\frac{2\pi nI}{r}$$

Where,

μ_0 = Permeability of free space

$= 4\pi \times 10^{-7}$ T m A^{-1}

$$|\mathbf{B}| = \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{2\pi \times 100 \times 0.4}{0.08}$$
$$= 3.14 \times 10^{-4} \text{ T}$$

Hence, the magnitude of the magnetic field is 3.14×10^{-4} T.

Question 4.2:

A long straight wire carries a current of 35 A. What is the magnitude of the field **B** at a point 20 cm from the wire?

Answer

Current in the wire, $I = 35$ A

Distance of a point from the wire, $r = 20$ cm $= 0.2$ m

Magnitude of the magnetic field at this point is given as:

$$B = \frac{\mu_0}{4\pi}\frac{2I}{r}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}$ T m A^{-1}

$$B = \frac{4\pi \times 10^{-7} \times 2 \times 35}{4\pi \times 0.2}$$
$$= 3.5 \times 10^{-5} \text{ T}$$

Hence, the magnitude of the magnetic field at a point 20 cm from the wire is 3.5×10^{-5} T.

Question 4.3:

A long straight wire in the horizontal plane carries a current of 50 A in north to south direction. Give the magnitude and direction of **B** at a point 2.5 m east of the wire.

Answer

Current in the wire, $I = 50$ A

A point is 2.5 m away from the East of the wire.

$\therefore$ Magnitude of the distance of the point from the wire, $r = 2.5$ m.

Magnitude of the magnetic field at that point is given by the relation, $B = \frac{\mu_0 2I}{4\pi r}$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}$ T m A^{-1}

$$B = \frac{4\pi \times 10^{-7} \times 2 \times 50}{4\pi \times 2.5}$$
$$= 4 \times 10^{-6} \text{ T}$$

The point is located normal to the wire length at a distance of 2.5 m. The direction of the current in the wire is vertically downward. Hence, according to the Maxwell's right hand thumb rule, the direction of the magnetic field at the given point is vertically upward.

Question 4.4:

A horizontal overhead power line carries a current of 90 A in east to west direction. What is the magnitude and direction of the magnetic field due to the current 1.5 m below the line?

Answer

Current in the power line, I = 90 A

Point is located below the power line at distance, r = 1.5 m

Hence, magnetic field at that point is given by the relation,

$$B = \frac{\mu_0 2I}{4\pi r}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}$ T m A^{-1}

$$B = \frac{4\pi \times 10^{-7} \times 2 \times 90}{4\pi \times 1.5} = 1.2 \times 10^{-5} \text{ T}$$

The current is flowing from East to West. The point is below the power line. Hence, according to Maxwell's right hand thumb rule, the direction of the magnetic field is towards the South.

Question 4.5:

What is the magnitude of magnetic force per unit length on a wire carrying a current of 8 A and making an angle of 30º with the direction of a uniform magnetic field of 0.15 T?

Answer

Current in the wire, $I = 8$ A

Magnitude of the uniform magnetic field, $B = 0.15$ T

Angle between the wire and magnetic field, $\theta = 30°$.

Magnetic force per unit length on the wire is given as:

$f = BI \sin\theta$

$= 0.15 \times 8 \times 1 \times \sin 30°$

$= 0.6 \text{ N m}^{-1}$

Hence, the magnetic force per unit length on the wire is 0.6 N m^{-1}.

Question 4.6:

A 3.0 cm wire carrying a current of 10 A is placed inside a solenoid perpendicular to its axis. The magnetic field inside the solenoid is given to be 0.27 T. What is the magnetic force on the wire?

Answer

Length of the wire, $l = 3$ cm $= 0.03$ m

Current flowing in the wire, $I = 10$ A

Magnetic field, $B = 0.27$ T

Angle between the current and magnetic field, $\theta = 90°$

Magnetic force exerted on the wire is given as:

$F = BIl\sin\theta$

$= 0.27 \times 10 \times 0.03 \sin 90°$

$= 8.1 \times 10^{-2}$ N

Hence, the magnetic force on the wire is 8.1×10^{-2} N. The direction of the force can be obtained from Fleming's left hand rule.

Question 4.7:

Two long and parallel straight wires A and B carrying currents of 8.0 A and 5.0 A in the same direction are separated by a distance of 4.0 cm. Estimate the force on a 10 cm section of wire A.

Answer

Current flowing in wire A, $I_A = 8.0$ A

Current flowing in wire B, $I_B = 5.0$ A

Distance between the two wires, $r = 4.0$ cm $= 0.04$ m

Length of a section of wire A, $l = 10$ cm $= 0.1$ m

Force exerted on length l due to the magnetic field is given as:

$$B = \frac{\mu_0 2\, I_A I_B l}{4\pi r}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}$ T m A^{-1}

$$B = \frac{4\pi \times 10^{-7} \times 2 \times 8 \times 5 \times 0.1}{4\pi \times 0.04}$$
$$= 2 \times 10^{-5} \text{ N}$$

The magnitude of force is 2×10^{-5} N. This is an attractive force normal to A towards B because the direction of the currents in the wires is the same.

Question 4.8:

A closely wound solenoid 80 cm long has 5 layers of windings of 400 turns each. The diameter of the solenoid is 1.8 cm. If the current carried is 8.0 A, estimate the magnitude of **B** inside the solenoid near its centre.

Answer

Length of the solenoid, $l = 80$ cm $= 0.8$ m

There are five layers of windings of 400 turns each on the solenoid.

∴ Total number of turns on the solenoid, $N = 5 \times 400 = 2000$

Diameter of the solenoid, $D = 1.8$ cm $= 0.018$ m

Current carried by the solenoid, $I = 8.0$ A

Magnitude of the magnetic field inside the solenoid near its centre is given by the relation,

$$B = \frac{\mu_0 N I}{l}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}$ T m A^{-1}

$$B = \frac{4\pi \times 10^{-7} \times 2000 \times 8}{0.8}$$
$$= 8\pi \times 10^{-3} = 2.512 \times 10^{-2} \text{ T}$$

Hence, the magnitude of the magnetic field inside the solenoid near its centre is 2.512×10^{-2} T.

Question 4.9:

A square coil of side 10 cm consists of 20 turns and carries a current of 12 A. The coil is suspended vertically and the normal to the plane of the coil makes an angle of 30º with the direction of a uniform horizontal magnetic field of magnitude 0.80 T. What is the magnitude of torque experienced by the coil?

Answer

Length of a side of the square coil, l = 10 cm = 0.1 m

Current flowing in the coil, I = 12 A

Number of turns on the coil, n = 20

Angle made by the plane of the coil with magnetic field, $\theta = 30°$

Strength of magnetic field, B = 0.80 T

Magnitude of the magnetic torque experienced by the coil in the magnetic field is given by the relation,

$\tau = n\ BIA \sin\theta$

Where,

A = Area of the square coil

$\Rightarrow l \times l = 0.1 \times 0.1 = 0.01\ \text{m}^2$

$\therefore \tau = 20 \times 0.8 \times 12 \times 0.01 \times \sin 30°$

= 0.96 N m

Hence, the magnitude of the torque experienced by the coil is 0.96 N m.

Question 4.10:

Two moving coil meters, M_1 and M_2 have the following particulars:

$R_1 = 10\ \Omega$, $N_1 = 30$,

$A_1 = 3.6 \times 10^{-3}\ \text{m}^2$, $B_1 = 0.25$ T

$R_2 = 14\ \Omega$, $N_2 = 42$,

$A_2 = 1.8 \times 10^{-3}\ \text{m}^2$, $B_2 = 0.50$ T

(The spring constants are identical for the two meters).

Determine the ratio of (a) current sensitivity and (b) voltage sensitivity of M_2 and M_1.

Answer

For moving coil meter M_1:

Resistance, $R_1 = 10\ \Omega$

Number of turns, $N_1 = 30$

Area of cross-section, $A_1 = 3.6 \times 10^{-3}\ \text{m}^2$

Magnetic field strength, $B_1 = 0.25$ T

Spring constant $K_1 = K$

For moving coil meter M_2:

Resistance, $R_2 = 14\ \Omega$

Number of turns, $N_2 = 42$

Area of cross-section, $A_2 = 1.8 \times 10^{-3}\ \text{m}^2$

Magnetic field strength, $B_2 = 0.50$ T

Spring constant, $K_2 = K$

Current sensitivity of M_1 is given as:

$$I_{s1} = \frac{N_1 \ B_1 \ A_1}{K_1}$$

And, current sensitivity of M_2 is given as:

$$I_{s2} = \frac{N_2 \ B_2 \ A_2}{K_2}$$

$\therefore$ Ratio $\frac{I_{s2}}{I_{s1}} = \frac{N_2 \ B_2 \ A_2 \ K_1}{K_2 \ N_1 \ B_1 \ A_1}$

$$= \frac{42 \times 0.5 \times 1.8 \times 10^{-3} \times K}{K \times 30 \times 0.25 \times 3.6 \times 10^{-3}} = 1.4$$

Hence, the ratio of current sensitivity of M_2 to M_1 is 1.4.

Voltage sensitivity for M_2 is given as:

$$V_{s2} = \frac{N_2 \ B_2 \ A_2}{K_2 \ R_2}$$

And, voltage sensitivity for M_1 is given as:

$$V_{s1} = \frac{N_1 \ B_1 \ A_1}{K_1}$$

$\therefore$ Ratio $\frac{V_{s2}}{Vs1} = \frac{N_2 \ B_2 \ A_2 \ K_1 \ R_1}{K_2 \ R_2 \ N_1 \ B_1 \ A_1}$

$$= \frac{42 \times 0.5 \times 1.8 \times 10^{-3} \times 10 \times K}{K \times 14 \times 30 \times 0.25 \times 3.6 \times 10^{-3}} = 1$$

Hence, the ratio of voltage sensitivity of M_2 to M_1 is 1.

Question 4.11:

In a chamber, a uniform magnetic field of 6.5 G (1 G = 10^{-4} T) is maintained. An electron is shot into the field with a speed of 4.8×10^6 m s^{-1} normal to the field. Explain why the path of the electron is a circle. Determine the radius of the circular orbit. ($e = 1.6 \times 10^{-19}$ C, $m_e = 9.1 \times 10^{-31}$ kg)

Answer

Magnetic field strength, $B = 6.5$ G $= 6.5 \times 10^{-4}$ T

Speed of the electron, $v = 4.8 \times 10^{6}$ m/s

Charge on the electron, $e = 1.6 \times 10^{-19}$ C

Mass of the electron, $m_e = 9.1 \times 10^{-31}$ kg

Angle between the shot electron and magnetic field, $\theta = 90°$

Magnetic force exerted on the electron in the magnetic field is given as:

$F = evB \sin\theta$

This force provides centripetal force to the moving electron. Hence, the electron starts moving in a circular path of radius r.

Hence, centripetal force exerted on the electron,

$$F_c = \frac{mv^2}{r}$$

In equilibrium, the centripetal force exerted on the electron is equal to the magnetic force i.e.,

$$F_c = F$$
$$\frac{mv^2}{r} = evB \sin\theta$$
$$r = \frac{mv}{Be\sin\theta}$$
$$= \frac{9.1\times10^{-31}\times4.8\times10^{6}}{6.5\times10^{-4}\times1.6\times10^{-19}\times\sin 90°}$$
$$= 4.2\times10^{-2} \text{ m } = 4.2 \text{ cm}$$

Hence, the radius of the circular orbit of the electron is 4.2 cm.

Question 4.12:

In Exercise 4.11 obtain the frequency of revolution of the electron in its circular orbit. Does the answer depend on the speed of the electron? Explain.

Answer

Magnetic field strength, $B = 6.5 \times 10^{-4}$ T

Charge of the electron, $e = 1.6 \times 10^{-19}$ C

Mass of the electron, $m_e = 9.1 \times 10^{-31}$ kg

Velocity of the electron, $v = 4.8 \times 10^{6}$ m/s

Radius of the orbit, $r = 4.2$ cm $= 0.042$ m

Frequency of revolution of the electron $= \nu$

Angular frequency of the electron $= \omega = 2\pi\nu$

Velocity of the electron is related to the angular frequency as:

$v = r\omega$

In the circular orbit, the magnetic force on the electron is balanced by the centripetal force. Hence, we can write:

$$evB = \frac{mv^2}{r}$$

$$eB = \frac{m}{r}(r\omega) = \frac{m}{r}(r2\pi\nu)$$

$$\nu = \frac{Be}{2\pi m}$$

This expression for frequency is independent of the speed of the electron.

On substituting the known values in this expression, we get the frequency as:

$$\nu = \frac{6.5\times10^{-4}\times1.6\times10^{-19}}{2\times3.14\times9.1\times10^{-31}}$$

$$= 18.2 \times 10^{6} \text{ Hz}$$

$$\approx 18 \text{ MHz}$$

Hence, the frequency of the electron is around 18 MHz and is independent of the speed of the electron.

Question 4.13:

A circular coil of 30 turns and radius 8.0 cm carrying a current of 6.0 A is suspended vertically in a uniform horizontal magnetic field of magnitude 1.0 T. The field lines make an angle of 60º with the normal of the coil. Calculate the magnitude of the counter torque that must be applied to prevent the coil from turning.

Would your answer change, if the circular coil in (a) were replaced by a planar coil of some irregular shape that encloses the same area? (All other particulars are also unaltered.)

Answer

Number of turns on the circular coil, $n = 30$

Radius of the coil, $r = 8.0$ cm $= 0.08$ m

Area of the coil $= \pi r^2 = \pi(0.08)^2 = 0.0201 \text{ m}^2$

Current flowing in the coil, $I = 6.0$ A

Magnetic field strength, $B = 1$ T

Angle between the field lines and normal with the coil surface,

$\theta = 60°$

The coil experiences a torque in the magnetic field. Hence, it turns. The counter torque applied to prevent the coil from turning is given by the relation,

$\tau = n\, IBA \sin\theta$ … (*i*)

$= 30 \times 6 \times 1 \times 0.0201 \times \sin 60°$

$= 3.133$ N m

It can be inferred from relation (*i*) that the magnitude of the applied torque is not dependent on the shape of the coil. It depends on the area of the coil. Hence, the answer would not change if the circular coil in the above case is replaced by a planar coil of some irregular shape that encloses the same area.

Question 4.14:

Two concentric circular coils X and Y of radii 16 cm and 10 cm, respectively, lie in the same vertical plane containing the north to south direction. Coil X has 20 turns and carries a current of 16 A; coil Y has 25 turns and carries a current of 18 A. The sense of the current in X is anticlockwise, and clockwise in Y, for an observer looking at the coils facing west. Give the magnitude and direction of the net magnetic field due to the coils at their centre.

Answer

Radius of coil X, r_1 = 16 cm = 0.16 m

Radius of coil Y, r_2 = 10 cm = 0.1 m

Number of turns of on coil X, n_1 = 20

Number of turns of on coil Y, n_2 = 25

Current in coil X, I_1 = 16 A

Current in coil Y, I_2 = 18 A

Magnetic field due to coil X at their centre is given by the relation,

$$B_1 = \frac{\mu_0 n_1 I_1}{2r_1}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}\ \mathrm{T\,m\,A^{-1}}$

$$\therefore B_1 = \frac{4\pi \times 10^{-7} \times 20 \times 16}{2 \times 0.16}$$
$$= 4\pi \times 10^{-4}\ \mathrm{T}\ \ \text{(towards East)}$$

Magnetic field due to coil Y at their centre is given by the relation,

$$B_2 = \frac{\mu_0 n_2 I_2}{2r_2}$$

$$= \frac{4\pi \times 10^{-7} \times 25 \times 18}{2 \times 0.10}$$

$$= 9\pi \times 10^{-4} \text{ T (towards West)}$$

Hence, net magnetic field can be obtained as:

$$B = B_2 - B_1$$

$$= 9\pi \times 10^{-4} - 4\pi \times 10^{-4}$$

$$= 5\pi \times 10^{-4} \text{ T}$$

$$= 1.57 \times 10^{-3} \text{ T (towards West)}$$

Question 4.15:

A magnetic field of 100 G (1 G = 10^{-4} T) is required which is uniform in a region of linear dimension about 10 cm and area of cross-section about 10^{-3} m^2. The maximum current-carrying capacity of a given coil of wire is 15 A and the number of turns per unit length that can be wound round a core is at most 1000 turns m^{-1}. Suggest some appropriate design particulars of a solenoid for the required purpose. Assume the core is not ferromagnetic

Answer

Magnetic field strength, B = 100 G = 100 × 10^{-4} T

Number of turns per unit length, n = 1000 turns m^{-1}

Current flowing in the coil, I = 15 A

Permeability of free space, $\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}

Magnetic field is given by the relation,

$$B = \mu_0 n I$$

$$\therefore nI = \frac{B}{\mu_0}$$

$$= \frac{100 \times 10^{-4}}{4\pi \times 10^{-7}} = 7957.74$$

$$\approx 8000 \text{ A/m}$$

If the length of the coil is taken as 50 cm, radius 4 cm, number of turns 400, and current 10 A, then these values are not unique for the given purpose. There is always a possibility of some adjustments with limits.

Question 4.16:

For a circular coil of radius *R* and *N* turns carrying current *I*, the magnitude of the magnetic field at a point on its axis at a distance *x* from its centre is given by,

$$B = \frac{\mu_0 I R^2 N}{2\left(x^2 + R^2\right)^{\frac{3}{2}}}$$

Show that this reduces to the familiar result for field at the centre of the coil.

Consider two parallel co-axial circular coils of equal radius *R*, and number of turns *N*, carrying equal currents in the same direction, and separated by a distance *R*. Show that the field on the axis around the mid-point between the coils is uniform over a distance that is small as compared to *R*, and is given by,

$$B = 0.72 - \frac{\mu_0 BNI}{R}$$, approximately.

[Such an arrangement to produce a nearly uniform magnetic field over a small region is known as *Helmholtz coils*.]

Answer

Radius of circular coil = *R*

Number of turns on the coil = *N*

Current in the coil = *I*

Magnetic field at a point on its axis at distance *x* is given by the relation,

$$B = \frac{\mu_0 I R^2 N}{2\left(x^2 + R^2\right)^{\frac{3}{2}}}$$

Where,

μ_0 = Permeability of free space

If the magnetic field at the centre of the coil is considered, then $x = 0$.

$$\therefore B = \frac{\mu_0 I R^2 N}{2R^3} = \frac{\mu_0 I N}{2R}$$

This is the familiar result for magnetic field at the centre of the coil.

Radii of two parallel co-axial circular coils = R

Number of turns on each coil = N

Current in both coils = I

Distance between both the coils = R

Let us consider point Q at distance d from the centre.

Then, one coil is at a distance of $\frac{R}{2} + d$ from point Q.

$\therefore$ Magnetic field at point Q is given as:

$$B_1 = \frac{\mu_0 N I R^2}{2\left[\left(\frac{R}{2} + d\right)^2 + R^2\right]^{\frac{3}{2}}}$$

Also, the other coil is at a distance of $\frac{R}{2} - d$ from point Q.

$\therefore$ Magnetic field due to this coil is given as:

$$B_2 = \frac{\mu_0 N I R^2}{2\left[\left(\frac{R}{2} - d\right)^2 + R^2\right]^{\frac{3}{2}}}$$

Total magnetic field,

$$B = B_1 + B_2$$

$$= \frac{\mu_0 I R^2}{2}\left[\left\{\left(\frac{R}{2}-d\right)^2 + R^2\right\}^{-\frac{3}{2}} + \left\{\left(\frac{R}{2}+d\right)^2 + R^2\right\}^{-\frac{3}{2}}\right]$$

$$= \frac{\mu_0 I R^2}{2}\left[\left(\frac{5R^2}{4} + d^2 - Rd\right)^{-\frac{3}{2}} + \left(\frac{5R^2}{4} + d^2 + Rd\right)^{-\frac{3}{2}}\right]$$

$$= \frac{\mu_0 I R^2}{2} \times \left(\frac{5R^2}{4}\right)^{-\frac{3}{2}}\left[\left(1 + \frac{4}{5}\frac{d^2}{R^2} - \frac{4}{5}\frac{d}{R}\right)^{-\frac{3}{2}} + \left(1 + \frac{4}{5}\frac{d^2}{R^2} + \frac{4}{5}\frac{d}{R}\right)^{-\frac{3}{2}}\right]$$

For $d << R$, neglecting the factor $\frac{d^2}{R^2}$, we get:

$$\approx \frac{\mu_0 I R^2}{2} \times \left(\frac{5R^2}{4}\right)^{-\frac{3}{2}} \times \left[\left(1 - \frac{4d}{5R}\right)^{-\frac{3}{2}} + \left(1 + \frac{4d}{5R}\right)^{-\frac{3}{2}}\right]$$

$$\approx \frac{\mu_0 I R^2 N}{2R^3} \times \left(\frac{4}{5}\right)^{\frac{3}{2}}\left[1 - \frac{6d}{5R} + 1 + \frac{6d}{5R}\right]$$

$$B = \left(\frac{4}{5}\right)^{\frac{3}{2}} \frac{\mu_0 I N}{R} = 0.72\left(\frac{\mu_0 I N}{R}\right)$$

Hence, it is proved that the field on the axis around the mid-point between the coils is uniform.

Question 4.17:

A toroid has a core (non-ferromagnetic) of inner radius 25 cm and outer radius 26 cm, around which 3500 turns of a wire are wound. If the current in the wire is 11 A, what is the magnetic field (a) outside the toroid, (b) inside the core of the toroid, and (c) in the empty space surrounded by the toroid.

Answer

Inner radius of the toroid, r_1 = 25 cm = 0.25 m

Outer radius of the toroid, r_2 = 26 cm = 0.26 m

Number of turns on the coil, N = 3500

Current in the coil, I = 11 A

Magnetic field outside a toroid is zero. It is non-zero only inside the core of a toroid.

Magnetic field inside the core of a toroid is given by the relation,

$$B = \frac{\mu_0 NI}{l}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}\ \mathrm{T\,m\,A^{-1}}$

l = length of toroid

$$= 2\pi\left[\frac{r_1 + r_2}{2}\right]$$
$$= \pi(0.25 + 0.26)$$
$$= 0.51\pi$$
$$\therefore B = \frac{4\pi \times 10^{-7} \times 3500 \times 11}{0.51\pi}$$
$$\approx 3.0 \times 10^{-2}\ \mathrm{T}$$

Magnetic field in the empty space surrounded by the toroid is zero.

Question 4.18:

Answer the following questions:

A magnetic field that varies in magnitude from point to point but has a constant direction (east to west) is set up in a chamber. A charged particle enters the chamber and travels undeflected along a straight path with constant speed. What can you say about the initial velocity of the particle?

A charged particle enters an environment of a strong and non-uniform magnetic field varying from point to point both in magnitude and direction, and comes out of it following a complicated trajectory. Would its final speed equal the initial speed if it suffered no collisions with the environment?

An electron travelling west to east enters a chamber having a uniform electrostatic field in north to south direction. Specify the direction in which a uniform magnetic field should be set up to prevent the electron from deflecting from its straight line path.

Answer

The initial velocity of the particle is either parallel or anti-parallel to the magnetic field. Hence, it travels along a straight path without suffering any deflection in the field.

Yes, the final speed of the charged particle will be equal to its initial speed. This is because magnetic force can change the direction of velocity, but not its magnitude.

An electron travelling from West to East enters a chamber having a uniform electrostatic field in the North-South direction. This moving electron can remain undeflected if the electric force acting on it is equal and opposite of magnetic field. Magnetic force is directed towards the South. According to Fleming's left hand rule, magnetic field should be applied in a vertically downward direction.

Question 4.19:

An electron emitted by a heated cathode and accelerated through a potential difference of 2.0 kV, enters a region with uniform magnetic field of 0.15 T. Determine the trajectory of the electron if the field (a) is transverse to its initial velocity, (b) makes an angle of 30º with the initial velocity.

Answer

Magnetic field strength, $B = 0.15$ T

Charge on the electron, $e = 1.6 \times 10^{-19}$ C

Mass of the electron, $m = 9.1 \times 10^{-31}$ kg

Potential difference, $V = 2.0$ kV $= 2 \times 10^3$ V

Thus, kinetic energy of the electron $= eV$

$$\Rightarrow eV = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2eV}{m}} \qquad \ldots (1)$$

Where,

v = velocity of the electron

Magnetic force on the electron provides the required centripetal force of the electron. Hence, the electron traces a circular path of radius r.

Magnetic force on the electron is given by the relation,

$B\ ev$

Centripetal force $= \frac{mv^2}{r}$

$$\therefore Bev = \frac{mv^2}{r}$$

$$r = \frac{mv}{Be} \qquad \ldots (2)$$

From equations (1) and (2), we get

$$r = \frac{m}{Be}\left[\frac{2eV}{m}\right]^{\frac{1}{2}}$$

$$= \frac{9.1\times10^{-31}}{0.15\times1.6\times10^{-19}}\times\left(\frac{2\times1.6\times10^{-19}\times2\times10^{3}}{9.1\times10^{-31}}\right)^{\frac{1}{2}}$$

$$= 100.55\times10^{-5}$$

$$= 1.01\times10^{-3}\ \text{m}$$

$$= 1\ \text{mm}$$

Hence, the electron has a circular trajectory of radius 1.0 mm normal to the magnetic field.

When the field makes an angle θ of 30° with initial velocity, the initial velocity will be,

$$v_1 = v\sin\theta$$

From equation (2), we can write the expression for new radius as:

$$r_1 = \frac{mv_1}{Be}$$

$$= \frac{mv\sin\theta}{Be}$$

$$= \frac{9.1\times10^{-31}}{0.15\times1.6\times10^{-19}}\times\left[\frac{2\times1.6\times10^{-19}\times2\times10^{3}}{9\times10^{-31}}\right]^{\frac{1}{2}}\times\sin 30^\circ$$

$$= 0.5\times10^{-3}\ \text{m}$$

$$= 0.5\ \text{mm}$$

Hence, the electron has a helical trajectory of radius 0.5 mm along the magnetic field direction.

Question 4.20:

A magnetic field set up using Helmholtz coils (described in Exercise 4.16) is uniform in a small region and has a magnitude of 0.75 T. In the same region, a uniform electrostatic field is maintained in a direction normal to the common axis of the coils. A narrow beam of (single species) charged particles all accelerated through 15 kV enters this region in a direction perpendicular to both the axis of the coils and the electrostatic field. If the beam remains undeflected when the electrostatic field is 9.0×10^{-5} V m^{-1}, make a simple guess as to what the beam contains. Why is the answer not unique?

Answer

Magnetic field, $B = 0.75$ T

Accelerating voltage, $V = 15$ kV $= 15 \times 10^3$ V

Electrostatic field, $E = 9 \times 10^5$ V m^{-1}

Mass of the electron $= m$

Charge of the electron $= e$

Velocity of the electron $= v$

Kinetic energy of the electron $= eV$

$$\Rightarrow \frac{1}{2}mv^2 = eV$$

$$\therefore \frac{e}{m} = \frac{v^2}{2V} \qquad \text{... (1)}$$

Since the particle remains undeflected by electric and magnetic fields, we can infer that the electric field is balancing the magnetic field.

$$\therefore eE = evB$$

$$v = \frac{E}{B} \qquad \text{... (2)}$$

Putting equation (2) in equation (1), we get

$$\frac{e}{m} = \frac{1}{2}\frac{\left(\frac{E}{B}\right)^2}{V} = \frac{E^2}{2VB^2}$$

$$= \frac{\left(9.0\times10^5\right)^2}{2\times15000\times\left(0.75\right)^2} = 4.8\times10^7 \text{ C/kg}$$

This value of specific charge *e*/*m* is equal to the value of deuteron or deuterium ions. This is not a unique answer. Other possible answers are He^{++}, Li^{++}, etc.

Question 4.21:

A straight horizontal conducting rod of length 0.45 m and mass 60 g is suspended by two vertical wires at its ends. A current of 5.0 A is set up in the rod through the wires.

What magnetic field should be set up normal to the conductor in order that the tension in the wires is zero?

What will be the total tension in the wires if the direction of current is reversed keeping the magnetic field same as before? (Ignore the mass of the wires.) g = 9.8 m s^{-2}.

Answer

Length of the rod, l = 0.45 m

Mass suspended by the wires, $m = 60$ g $= 60 \times 10^{-3}$ kg

Acceleration due to gravity, g = 9.8 m/s^2

Current in the rod flowing through the wire, $I = 5$ A

Magnetic field (B) is equal and opposite to the weight of the wire i.e.,

$$BIl = mg$$

$$\therefore B = \frac{mg}{Il}$$

$$= \frac{60 \times 10^{-3} \times 9.8}{5 \times 0.45} = 0.26 \text{ T}$$

A horizontal magnetic field of 0.26 T normal to the length of the conductor should be set up in order to get zero tension in the wire. The magnetic field should be such that Fleming's left hand rule gives an upward magnetic force.

If the direction of the current is revered, then the force due to magnetic field and the weight of the wire acts in a vertically downward direction.

$\therefore$Total tension in the wire = $BIl + mg$

$$= 0.26 \times 5 \times 0.45 + \left(60 \times 10^{-3}\right) \times 9.8$$

$$= 1.176 \text{ N}$$

Question 4.22:

The wires which connect the battery of an automobile to its starting motor carry a current of 300 A (for a short time). What is the force per unit length between the wires if they are 70 cm long and 1.5 cm apart? Is the force attractive or repulsive?

Answer

Current in both wires, $I = 300$ A

Distance between the wires, $r = 1.5$ cm $= 0.015$ m

Length of the two wires, $l = 70$ cm $= 0.7$ m

Force between the two wires is given by the relation,

$$F = \frac{\mu_0 I^2}{2\pi r}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}\ \text{T m A}^{-1}$

$$\therefore F = \frac{4\pi \times 10^{-7} \times (300)^2}{2\pi \times 0.015}$$
$$= 1.2\ \text{N/m}$$

Since the direction of the current in the wires is opposite, a repulsive force exists between them.

Question 4.23:

A uniform magnetic field of 1.5 T exists in a cylindrical region of radius10.0 cm, its direction parallel to the axis along east to west. A wire carrying current of 7.0 A in the north to south direction passes through this region. What is the magnitude and direction of the force on the wire if,

the wire intersects the axis,

the wire is turned from N-S to northeast-northwest direction,

the wire in the N-S direction is lowered from the axis by a distance of 6.0 cm?

Answer

Magnetic field strength, B = 1.5 T

Radius of the cylindrical region, r = 10 cm = 0.1 m

Current in the wire passing through the cylindrical region, I = 7 A

If the wire intersects the axis, then the length of the wire is the diameter of the cylindrical region.

Thus, $l = 2r = 0.2$ m

Angle between magnetic field and current, $\theta = 90°$

Magnetic force acting on the wire is given by the relation,

$F = BIl \sin\theta$

$= 1.5 \times 7 \times 0.2 \times \sin 90°$

$= 2.1$ N

Hence, a force of 2.1 N acts on the wire in a vertically downward direction.

New length of the wire after turning it to the Northeast-Northwest direction can be given as: :

$$l_1 = \frac{l}{\sin\theta}$$

Angle between magnetic field and current, $\theta = 45°$

Force on the wire,

$F = BIl_1 \sin\theta$

$$\begin{aligned} &= BIl \\ &= 1.5 \times 7 \times 0.2 \\ &= 2.1 \text{ N} \end{aligned}$$

Hence, a force of 2.1 N acts vertically downward on the wire. This is independent of angleθbecause $l \sin\theta$ is fixed.

The wire is lowered from the axis by distance, $d = 6.0$ cm

Let l_2 be the new length of the wire.

$$\begin{aligned} \therefore \left(\frac{l_2}{2}\right)^2 &= 4(d+r) \\ &= 4(10+6) = 4(16) \\ \therefore l_2 &= 8 \times 2 = 16 \text{ cm} = 0.16 \text{ m} \end{aligned}$$

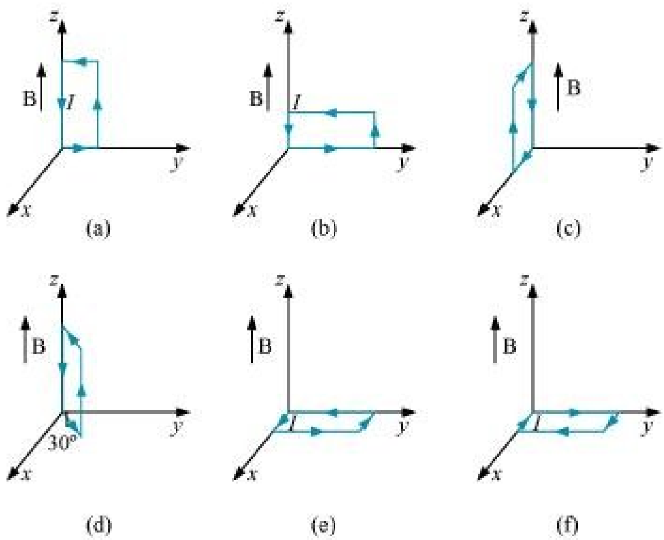
Magnetic force exerted on the wire,

$$F_2 = BIl_2$$
$$= 1.5 \times 7 \times 0.16$$
$$= 1.68 \text{ N}$$

Hence, a force of 1.68 N acts in a vertically downward direction on the wire.

Question 4.24:

A uniform magnetic field of 3000 G is established along the positive *z*-direction. A rectangular loop of sides 10 cm and 5 cm carries a current of 12 A. What is the torque on the loop in the different cases shown in Fig. 4.28? What is the force on each case? Which case corresponds to stable equilibrium?



Answer

Magnetic field strength, $B = 3000 \text{ G} = 3000 \times 10^{-4} \text{ T} = 0.3 \text{ T}$

Length of the rectangular loop, $l = 10$ cm

Width of the rectangular loop, $b = 5$ cm

Area of the loop,

$A = l \times b = 10 \times 5 = 50 \text{ cm}^2 = 50 \times 10^{-4} \text{ m}^2$

Current in the loop, $I = 12$ A

Now, taking the anti-clockwise direction of the current as positive and vise-versa:

Torque, $\vec{\tau} = I\vec{A} \times \vec{B}$

From the given figure, it can be observed that A is normal to the y-z plane and B is directed along the z-axis.

$$\therefore \tau = 12 \times \left(50 \times 10^{-4}\right)\hat{i} \times 0.3\hat{k}$$
$$= -1.8 \times 10^{-2}\,\hat{j} \text{ N m}$$

The torque is 1.8×10^{-2} N m along the negative y-direction. The force on the loop is zero because the angle between A and B is zero.

This case is similar to case (a). Hence, the answer is the same as (a).

Torque $\tau = I\vec{A} \times \vec{B}$

From the given figure, it can be observed that A is normal to the x-z plane and B is directed along the z-axis.

$$\therefore \tau = -12 \times \left(50 \times 10^{-4}\right)\hat{j} \times 0.3\hat{k}$$
$$= -1.8 \times 10^{-2}\hat{i} \text{ N m}$$

The torque is 1.8×10^{-2} N m along the negative x direction and the force is zero.

Magnitude of torque is given as:

$$|\tau| = IAB$$
$$= 12 \times 50 \times 10^{-4} \times 0.3$$
$$= 1.8 \times 10^{-2} \text{ N m}$$

Torque is 1.8×10^{-2} N m at an angle of 240° with positive x direction. The force is zero.

Torque $\tau = I\vec{A} \times \vec{B}$

$$= \left(50 \times 10^{-4} \times 12\right)\hat{k} \times 0.3\hat{k}$$
$$= 0$$

Hence, the torque is zero. The force is also zero.

Torque $\tau = I\vec{A} \times \vec{B}$

$$= \left(50 \times 10^{-4} \times 12\right)\hat{k} \times 0.3\hat{k}$$
$$= 0$$

Hence, the torque is zero. The force is also zero.

In case (e), the direction of $I\vec{A}$ and $\vec{B}$ is the same and the angle between them is zero. If displaced, they come back to an equilibrium. Hence, its equilibrium is stable.

Whereas, in case (f), the direction of $I\vec{A}$ and $\vec{B}$ is opposite. The angle between them is 180°. If disturbed, it does not come back to its original position. Hence, its equilibrium is unstable.

Question 4.25:

A circular coil of 20 turns and radius 10 cm is placed in a uniform magnetic field of 0.10 T normal to the plane of the coil. If the current in the coil is 5.0 A, what is the

total torque on the coil,

total force on the coil,

average force on each electron in the coil due to the magnetic field?

(The coil is made of copper wire of cross-sectional area 10^{-5} m^2, and the free electron density in copper is given to be about 10^{29} m^{-3}.)

Answer

Number of turns on the circular coil, $n = 20$

Radius of the coil, $r = 10$ cm $= 0.1$ m

Magnetic field strength, $B = 0.10$ T

Current in the coil, $I = 5.0$ A

The total torque on the coil is zero because the field is uniform.

The total force on the coil is zero because the field is uniform.

Cross-sectional area of copper coil, $A = 10^{-5}$ m^2

Number of free electrons per cubic meter in copper, $N = 10^{29}$ /m^3

Charge on the electron, $e = 1.6 \times 10^{-19}$ C

Magnetic force, $F = Bev_d$

Where,

v_d = Drift velocity of electrons

$$= \frac{I}{NeA}$$

$$\therefore F = \frac{BeI}{NeA}$$

$$= \frac{0.10 \times 5.0}{10^{29} \times 10^{-5}} = 5 \times 10^{-25} \text{ N}$$

Hence, the average force on each electron is 5×10^{-25} N.

Question 4.26:

A solenoid 60 cm long and of radius 4.0 cm has 3 layers of windings of 300 turns each. A 2.0 cm long wire of mass 2.5 g lies inside the solenoid (near its centre) normal to its axis; both the wire and the axis of the solenoid are in the horizontal plane. The wire is connected through two leads parallel to the axis of the solenoid to an external battery which supplies a current of 6.0 A in the wire. What value of current (with appropriate sense of circulation) in the windings of the solenoid can support the weight of the wire? $g = 9.8$ m s^{-2}

Answer

Length of the solenoid, L = 60 cm = 0.6 m

Radius of the solenoid, r = 4.0 cm = 0.04 m

It is given that there are 3 layers of windings of 300 turns each.

$\therefore$ Total number of turns, $n = 3 \times 300 = 900$

Length of the wire, $l = 2$ cm $= 0.02$ m

Mass of the wire, $m = 2.5$ g $= 2.5 \times 10^{-3}$ kg

Current flowing through the wire, $i = 6$ A

Acceleration due to gravity, g = 9.8 m/s^2

Magnetic field produced inside the solenoid, $B = \frac{\mu_0 n I}{L}$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}\ \text{T m A}^{-1}$

I = Current flowing through the windings of the solenoid

Magnetic force is given by the relation,

$$F = Bil$$
$$= \frac{\mu_0 n I}{L} il$$

Also, the force on the wire is equal to the weight of the wire.

$$\therefore mg = \frac{\mu_0 n I i l}{L}$$
$$I = \frac{mgL}{\mu_0 n i l}$$
$$= \frac{2.5 \times 10^{-3} \times 9.8 \times 0.6}{4\pi \times 10^{-7} \times 900 \times 0.02 \times 6} = 108 \text{ A}$$

Hence, the current flowing through the solenoid is 108 A.

Question 4.27:

A galvanometer coil has a resistance of 12 Ω and the metre shows full scale deflection for a current of 3 mA. How will you convert the metre into a voltmeter of range 0 to 18 V?

Answer

Resistance of the galvanometer coil, $G = 12\ \Omega$

Current for which there is full scale deflection, $I_g = 3\text{ mA} = 3 \times 10^{-3}\text{ A}$

Range of the voltmeter is 0, which needs to be converted to 18 V.

$\therefore V = 18\text{ V}$

Let a resistor of resistance R be connected in series with the galvanometer to convert it into a voltmeter. This resistance is given as:

$$R = \frac{V}{I_g} - \mathrm{G}$$

$$= \frac{18}{3 \times 10^{-3}} - 12 = 6000 - 12 = 5988\ \Omega$$

Hence, a resistor of resistance $5988\ \Omega$ is to be connected in series with the galvanometer.

Question 4.28:

A galvanometer coil has a resistance of 15 Ω and the metre shows full scale deflection for a current of 4 mA. How will you convert the metre into an ammeter of range 0 to 6 A?

Answer

Resistance of the galvanometer coil, $G = 15\ \Omega$

Current for which the galvanometer shows full scale deflection,

$I_g = 4\text{ mA} = 4 \times 10^{-3}\text{ A}$

Range of the ammeter is 0, which needs to be converted to 6 A.

$\therefore$ Current, $I = 6\text{ A}$

A shunt resistor of resistance S is to be connected in parallel with the galvanometer to convert it into an ammeter. The value of S is given as:

$$S = \frac{I_g G}{I - I_g}$$

$$= \frac{4 \times 10^{-3} \times 15}{6 - 4 \times 10^{-3}}$$

$$S = \frac{6 \times 10^{-2}}{6 - 0.004} = \frac{0.06}{5.996}$$

$$\approx 0.01\ \Omega = 10\ \text{m}\Omega$$

Hence, a $10\ \text{m}\Omega$ shunt resistor is to be connected in parallel with the galvanometer.