CBSE Class 8 Maths Notes Chapter 1 Rational Numbers

Rational Number: A number is called rational if we can write the number in the form of $\frac{p}{q}$, where p and q are integers and q ≠ 0

i.e., $1 = \frac{1}{1}$, $2 = \frac{2}{1}$, $0 = \frac{0}{1}$ and $\frac{5}{8}$, $\frac{-3}{14}$, $\frac{7}{-15}$ are all rational numbers.

Between two rational numbers x and y, there exists a rational number $\frac{x+y}{2}$

We can find countless rational numbers between two rational numbers.

$\frac{-x}{y}$ is called the additive inverse of $\frac{x}{y}$ and vice-versa.

$\frac{y}{x}$ is called the multiplicative inverse or reciprocal of $\frac{x}{y}$.

Rational number 0 is the additive identity for all rational numbers because a number does not change when 0 is added to it.

Subtraction:

$$\frac{2}{5} - \frac{5}{6} = \frac{12-25}{30} = \frac{-13}{30}$$

$$\frac{5}{6} - \frac{2}{5} = \frac{25-12}{20} = \frac{13}{30}$$

Hence, subtraction is not associative for rational numbers.

(iii) Multiplication: Multiplication is commutative for rational numbers. In general, a × b = b × a, for any two rational numbers a and b.

$$\frac{-3}{4} \times \frac{5}{6} = \frac{5}{6} \times \left(\frac{-3}{4}\right) = \frac{-15}{24}$$

Division:

$$\frac{-3}{7} \div \frac{2}{5} = \frac{-3}{7} \times \frac{5}{2} = \frac{-15}{14}$$

$$\frac{2}{5} \div \left(\frac{-3}{7}\right) = \frac{2}{5} \times \frac{7}{-3} = \frac{14}{-15}$$

$$\frac{-3}{7} \div \frac{2}{5} \neq \frac{2}{5} \div \left(\frac{-3}{7}\right)$$

Hence, division is not Cumulative for rational numbers.

(iii) Associativity:

Addition:

$$eg. \ \frac{-2}{5} + \left[\frac{3}{4} + \left(\frac{-7}{8}\right)\right] = \frac{-2}{5} + \left(\frac{-1}{8}\right) = \frac{-21}{40} = \left[\frac{-2}{5} + \frac{3}{4}\right] + \left(\frac{-7}{8}\right) = \frac{7}{20} - \frac{7}{8} = \frac{-21}{40}$$

So, addition is associative for rational numbers, i.e., for any three rational numbers a, b and c, a + (b + c) = (a + b) + c.

Hence, subtraction is not associative for rational numbers.

Multiplication:

eg.

$$\frac{-2}{3} \times \left(\frac{2}{5} \times \frac{6}{7}\right) = \frac{-2}{3} \times \frac{12}{35} = \frac{-24}{105} = \frac{-8}{35}$$

and

$$\left(\frac{-2}{3} \times \frac{2}{5}\right) \times \frac{6}{7} = \frac{-4}{15} \times \frac{6}{7} = \frac{-24}{105} = \frac{-8}{35}$$

So, multiplication is associative for rational number, i.e., for any three rational numbers a, b and c, a × (b × c) = (a × b) × c.

Division:

eg.

$$\frac{2}{3} \div \left[\frac{-1}{2} \div \frac{2}{5}\right] = \left[\frac{2}{3} \div \left(\frac{-1}{2}\right)\right] \div \frac{2}{5}$$

We have, LHS

$$\frac{2}{3} \div \left[-\frac{1}{2} \div \frac{2}{5}\right] = \left[\frac{2}{3} \div \frac{1}{2} \times \frac{5}{2}\right]$$

$$= \frac{2}{3} \div \left(\frac{-5}{4}\right) = \frac{2}{3} \times \frac{4}{-5} = \frac{8}{-15}$$

RHS

$$\left[\frac{2}{3} \div \left(\frac{-1}{2}\right)\right] \div \frac{2}{5} = \left[\frac{2}{3} \times \left(\frac{-2}{1}\right)\right] \div \frac{2}{5} = \left(\frac{-4}{3}\right) \div \frac{2}{5}$$

$$= \frac{-4}{3} \times \frac{5}{2} = \frac{-20}{6} = \frac{-10}{3}$$

But   LHS ≠ RHS

Hence, the division is not associative for rational numbers.

Distributivity of multiplication over addition for rational number : For all rational numbers a, b and c, a(b + c) = ab + ac

Also, $\dfrac{-2}{5} \times \dfrac{3}{8} = \dfrac{-6}{40} = \dfrac{-3}{20}$

and $\dfrac{-2}{5} \times \dfrac{-3}{4} = \dfrac{6}{20}$

Therefore, $\left(\dfrac{-2}{5} \times \dfrac{3}{8}\right) + \left(\dfrac{-2}{5} \times \dfrac{-3}{4}\right) = \dfrac{-3}{20} + \dfrac{6}{20} = \dfrac{3}{20}$

Thus, $\dfrac{2}{5} \times \left\{\dfrac{3}{8} + \left(\dfrac{-3}{4}\right)\right\} = \left(\dfrac{-2}{5} \times \dfrac{3}{8}\right) + \left\{\dfrac{-2}{5} \times \left(\dfrac{-3}{4}\right)\right\}$

Distributivity of multiplication over subtraction for rational number: For any three rational numbers a, b and c, a (b − c) = ab − ac

eg. Let $\dfrac{1}{2}, \dfrac{-2}{5}$ an $\dfrac{-3}{10}$ are any three rational numbers, then

$$\dfrac{1}{2} \times \left\{\dfrac{-2}{5} - \left(\dfrac{-3}{10}\right)\right\} = \dfrac{1}{2} \times \left\{\dfrac{-4 - (-3)}{10}\right\} = \dfrac{1}{2} \times \left(\dfrac{-4 + 3}{10}\right)$$

$$= \dfrac{1}{2} \times \left(\dfrac{-1}{10}\right) = \dfrac{-1}{20}$$

Also, $\dfrac{1}{2} \times \left(\dfrac{-2}{5}\right) = \dfrac{-2}{10} = -\dfrac{1}{5}$

and $\dfrac{1}{2} \times \left(\dfrac{-3}{10}\right) = \dfrac{-3}{10}$

Therefore, $\left(\dfrac{1}{2} \times \dfrac{-2}{5}\right) - \left(\dfrac{1}{2} \times \dfrac{-3}{10}\right) = \dfrac{-1}{5} - \left(\dfrac{-3}{20}\right)$

$$= \dfrac{-1}{5} + \dfrac{3}{20}$$

$$= \dfrac{-4 + 3}{20} = \dfrac{-1}{20}$$

Thus, $\dfrac{1}{2} \times \left\{\dfrac{-2}{5} - \left(\dfrac{-3}{10}\right)\right\} = \left(\dfrac{1}{2} \times \dfrac{-2}{5}\right) - \left(\dfrac{1}{2} \times \dfrac{-3}{10}\right)$

The numbers 1, 2, 3, 4, ... are called natural numbers.

If we add 0 to the collection of natural numbers, what we get is called the cc

equations.

Properties of Rational Numbers

The list of properties of rational numbers can be given as follows:

- Closure
- Commutativity
- Associativity
- The role of zero (0)
- The role of 1
- Negative of a number
- Reciprocal
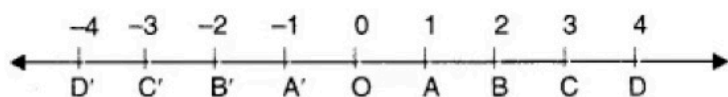- Distributivity of multiplication over addition for rational numbers.

Distributivity of Multiplication Over Addition for Rational Numbers
For all rational numbers a, b and c,
a(b + c) = ab + ac
a(b − c) = ab − ac.

Representation of Rational Numbers on the Number Line

- We draw a line.
- We mark a point O on it and name it 0. Mark a point to the right of 0. Name it 1. The distance between these two points is called unit distance.
- Mark a point to the right of 1 at unit distance and name it 2.
- Proceeding in this manner, we can mark points 3, 4, 5,
- Similarly we can mark − 1, − 2, − 3, − 4, − 5, ………. to the left of 0. This line is called the number line.
- This line extends indefinitely on both sides.

The positive rational numbers are represented by points on the number line to the right of O whereas the negative rational numbers are represented by points on the number line to the left of O.

EXERCISE 1.1

1. Using appropriate properties find.

   (i) $-\dfrac{2}{3} \times \dfrac{3}{5} + \dfrac{5}{2} - \dfrac{3}{5} \times \dfrac{1}{6}$

   (ii) $\dfrac{2}{5} \times \left(-\dfrac{3}{7}\right) - \dfrac{1}{6} \times \dfrac{3}{2} + \dfrac{1}{14} \times \dfrac{2}{5}$

2. Write the additive inverse of each of the following.

   (i) $\dfrac{2}{8}$ (ii) $\dfrac{-5}{9}$ (iii) $\dfrac{-6}{-5}$ (iv) $\dfrac{2}{-9}$ (v) $\dfrac{19}{-6}$

3. Verify that $-(-x) = x$ for.

   (i) $x = \dfrac{11}{15}$ (ii) $x = -\dfrac{13}{17}$

4. Find the multiplicative inverse of the following.

   (i) -13 (ii) $\dfrac{-13}{19}$ (iii) $\dfrac{1}{5}$ (iv) $\dfrac{-5}{8} \times \dfrac{-3}{7}$

   (v) $-1 \times \dfrac{-2}{5}$ (vi) -1

5. Name the property under multiplication used in each of the following.

   (i) $\dfrac{-4}{5} \times 1 = 1 \times \dfrac{-4}{5} = -\dfrac{4}{5}$

   (ii) $-\dfrac{13}{17} \times \dfrac{-2}{7} = \dfrac{-2}{7} \times \dfrac{-13}{17}$

   (iii) $\dfrac{-19}{29} \times \dfrac{29}{-19} = 1$

6. Multiply $\dfrac{6}{13}$ by the reciprocal of $\dfrac{-7}{16}$.

7. Tell what property allows you to compute $\dfrac{1}{3} \times \left(6 \times \dfrac{4}{3}\right)$ as $\left(\dfrac{1}{3} \times 6\right) \times \dfrac{4}{3}$.

8. Is $\dfrac{8}{9}$ the multiplicative inverse of $-1\dfrac{1}{8}$? Why or why not?

9. Is 0.3 the multiplicative inverse of $3\dfrac{1}{3}$? Why or why not?

10. Write.

    (i) The rational number that does not have a reciprocal.

    (ii) The rational numbers that are equal to their reciprocals.

    (iii) The rational number that is equal to its negative.

11. Fill in the blanks.

    (i) Zero has _____ reciprocal.

    (ii) The numbers _____ and _____ are their own reciprocals

    (iii) The reciprocal of -5 is _____.

    (iv) Reciprocal of $\dfrac{1}{x}$, where $x \neq 0$ is _____.

    (v) The product of two rational numbers is always a _____.

    (vi) The reciprocal of a positive rational number is _____.

1. Represent these numbers on the number line. (i) $\dfrac{7}{4}$ (ii) $\dfrac{-5}{6}$

2. Represent $\dfrac{-2}{11}, \dfrac{-5}{11}, \dfrac{-9}{11}$ on the number line.

3. Write five rational numbers which are smaller than 2.

4. Find ten rational numbers between $\dfrac{-2}{5}$ and $\dfrac{1}{2}$.

5. Find five rational numbers between.

    (i) $\dfrac{2}{3}$ and $\dfrac{4}{5}$ (ii) $\dfrac{-3}{2}$ and $\dfrac{5}{3}$ (iii) $\dfrac{1}{4}$ and $\dfrac{1}{2}$

6. Write five rational numbers greater than –2.

7. Find ten rational numbers between $\dfrac{3}{5}$ and $\dfrac{3}{4}$.

EXERCISE 2.1

Solve the following equations.

1. $x - 2 = 7$ 2. $y + 3 = 10$ 3. $6 = z + 2$

4. $\dfrac{3}{7} + x = \dfrac{17}{7}$ 5. $6x = 12$ 6. $\dfrac{t}{5} = 10$

7. $\dfrac{2x}{3} = 18$ 8. $1.6 = \dfrac{y}{1.5}$ 9. $7x - 9 = 16$

10. $14y - 8 = 13$ 11. $17 + 6p = 9$ 12. $\dfrac{x}{3} + 1 = \dfrac{7}{15}$

1. If you subtract $\frac{1}{2}$ from a number and multiply the result by $\frac{1}{2}$, you get $\frac{1}{8}$. What is the number?

2. The perimeter of a rectangular swimming pool is 154 m. Its length is 2 m more than twice its breadth. What are the length and the breadth of the pool?

3. The base of an isosceles triangle is $\frac{4}{3}$ cm. The perimeter of the triangle is $4\frac{2}{15}$ cm. What is the length of either of the remaining equal sides?

4. Sum of two numbers is 95. If one exceeds the other by 15, find the numbers.

5. Two numbers are in the ratio 5:3. If they differ by 18, what are the numbers?

6. Three consecutive integers add up to 51. What are these integers?

7. The sum of three consecutive multiples of 8 is 888. Find the multiples.

8. Three consecutive integers are such that when they are taken in increasing order and multiplied by 2, 3 and 4 respectively, they add up to 74. Find these numbers.

9. The ages of Rahul and Haroon are in the ratio 5:7. Four years later the sum of their ages will be 56 years. What are their present ages?

10. The number of boys and girls in a class are in the ratio 7:5. The number of boys is 8 more than the number of girls. What is the total class strength?

11. Baichung's father is 26 years younger than Baichung's grandfather and 29 years older than Baichung. The sum of the ages of all the three is 135 years. What is the age of each one of them?

12. Fifteen years from now Ravi's age will be four times his present age. What is Ravi's present age?

13. A rational number is such that when you multiply it by $\frac{5}{2}$ and add $\frac{2}{3}$ to the product, you get $-\frac{7}{12}$. What is the number?

14. Lakshmi is a cashier in a bank. She has currency notes of denominations ₹ 100, ₹ 50 and ₹ 10, respectively. The ratio of the number of these notes is 2:3:5. The total cash with Lakshmi is ₹ 4,00,000. How many notes of each denomination does she have?

15. I have a total of ₹ 300 in coins of denomination ₹ 1, ₹ 2 and ₹ 5. The number of ₹ 2 coins is 3 times the number of ₹ 5 coins. The total number of coins is 160. How many coins of each denomination are with me?

16. The organisers of an essay competition decide that a winner in the competition gets a prize of ₹ 100 and a participant who does not win gets a prize of ₹ 25. The total prize money distributed is ₹ 3,000. Find the number of winners, if the total number of participants is 63.

Solve the following equations and check your results.

1. $3x = 2x + 18$

2. $5t - 3 = 3t - 5$

3. $5x + 9 = 5 + 3x$

4. $4z + 3 = 6 + 2z$

5. $2x - 1 = 14 - x$

6. $8x + 4 = 3 (x - 1) + 7$

7. $x = \frac{4}{5} (x + 10)$

8. $\frac{2x}{3} + 1 = \frac{7x}{15} + 3$

9. $2y + \frac{5}{3} = \frac{26}{3} - y$

10. $3m = 5 m - \frac{8}{5}$

EXERCISE 2.4

1. Amina thinks of a number and subtracts $\dfrac{5}{2}$ from it. She multiplies the result by 8. The result now obtained is 3 times the same number she thought of. What is the number?

2. A positive number is 5 times another number. If 21 is added to both the numbers, then one of the new numbers becomes twice the other new number. What are the numbers?

3. Sum of the digits of a two-digit number is 9. When we interchange the digits, it is found that the resulting new number is greater than the original number by 27. What is the two-digit number?

4. One of the two digits of a two digit number is three times the other digit. If you interchange the digits of this two-digit number and add the resulting number to the original number, you get 88. What is the original number?

5. Shobo's mother's present age is six times Shobo's present age. Shobo's age five years from now will be one third of his mother's present age. What are their present ages?

6. There is a narrow rectangular plot, reserved for a school, in Mahuli village. The length and breadth of the plot are in the ratio 11:4. At the rate ₹100 per metre it will cost the village panchayat ₹75000 to fence the plot. What are the dimensions of the plot?

7. Hasan buys two kinds of cloth materials for school uniforms, shirt material that costs him ₹ 50 per metre and trouser material that costs him ₹ 90 per metre.