Introduction to Linear Equations

A **linear equation** is an algebraic equation in which each term is either a constant or the product of a constant and a variable. This variable is always single power form. A simple example of a linear equation with only one variable, x, may be written in the form: ax + b = 0, where a and b are constants and a ≠ 0.

These equations are called**Linear Equations in one variable**. Examples:

- 2x
- 5/4 (x-4)
- 3y-7
- $x^2 + 1$ is an non-linear as the variable has power more than 1.
- 5xy + 10 is not a linear equation in one variable as there are two variables

For example, if a child has 2 chocolates of 10 rupees and a 20 rupee note, the total value can be calculated using the equation 2x + 20 = 2 * 10 + 20 = 40. Here 'x' is variable and is the value of the chocolate.

Linear equation in 1 variable
2x + 20

Another example, one child buys three toys (costing rupees 20 each) from a shopkeeper but gives only 5 rupees to him. The total value of items with child will be 3y - 5, where y is the cost of one toy.

Linear equation in 1 variable
3y - 5

Class 8 Maths Linear Equations in One Variable Algebraic Equations and Solutions

Algebraic Equations and Solutions

An **algebraic equation** is an equality involving variables. The expression on the left of equality is called **LHS (Left Hand Side)** and on the right is called **RHS (Right Hand Side)**.

- From the above example, an algebraic equation is 2x + 20 = 40, where x is variable. Here 2x + 20 is LHS and 40 is RHS.

A **Solution** is the true value of the variables in the algebraic equations for which the equation holds true and LHS is equal to RHS.

- From the above example, algebraic equation is 2x + 20 = 40.
  - For x = 10, 2 * 10 + 20 = 40. Or, 40 = 40. Since LHS = RHS, hence x = 10 is the solution.
  - For x = 5, 2 * 5 + 20 = 40.Or, 30 = 40. Since LHS ≠ RHS, hence x = 5 is not the solution.

Solving Equations having linear expression/equation on one side and number on the other

There are various methods to solve the algebraic equations:

1. By addition/subtraction and multiplication/division

Example, an algebraic equation is 2x - 3 = 7

Adding 3 both sides, 2x - 3 + 3 = 7 + 3 or 2x = 10

Dividing both sides by 2, 2x/2 = 10/2 or**x = 5**

2. By transposing

Example, an algebraic equation is x - 3 = 7

Transposing 3 to RHS, x = 7 + 3 o**x = 10**

3. Using combination of above 2 methods

Example, an algebraic equation is x/3 + 5/2 = -3/2

Transposing 5/2 to RHS, x/3 = -3/2- 5/2 or x/3= -8/2 or x/3 = -4

Multiplying both sides by 3, x/3 * 3 = -4 * 3 o**x = -12**

Suppose there are two children whose ages are unknown. However, there are 2 conditions related to the age are known:

1. Suppose the boy is 2 years older than the girl.
2. Sum of their ages is 14.

Suppose the age of the girl is x.
Age of the boy = x + 2
Sum of their ages = x + x +2 = 14
So, 2x + 2 = 14
Or 2x = 12 (transposing 2 to RHS)
Or x = 6 (Dividing both sides by 2)

Hence, age of the girl is 6 and the age of boy is 6 + 2 = 8.
So the sum of their ages is 8 + 6 = 14.

Example

Problem: Solve the following equations:

(i) 6 = z + 2 (ii) $\frac{3}{7} + x = \frac{17}{7}$ (iii) $1.6 = \frac{y}{1.5}$ (iv) $\frac{x}{3} + 1 = \frac{7}{15}$

Solution:

(i) 6 = z + 2
Subtracting 2 from both sides,
6-2 = z + 2 -2
4 = z or z = 4

(ii) $\frac{3}{7} + x = \frac{17}{7}$
Transposing $\frac{3}{7}$ to RHS,
$x = \frac{17}{7} - \frac{3}{7}$
$x = \frac{17-3}{7} = \frac{14}{7} = 2$

(iii) $1.6 = \frac{y}{1.5}$
Multiplying both sides by 1.5,
$1.6 * 1.5 = \frac{y}{1.5} * 1.5$
2.4 = y or y = 2.4

(iv) $\frac{x}{3} + 1 = \frac{7}{15}$
Transposing 1 to RHS,
$\frac{x}{3} = \frac{7}{15} - 1$
$\frac{x}{3} = \frac{7-15}{15} = \frac{-8}{15}$
Multiplying both sides by 3,
$\frac{x}{3} * 3 = \frac{-8}{15} * 3$
$x = \frac{-8}{5}$

Example

Problem: The perimeter of a rectangular swimming pool is 154 m. Its length is 2 m more than twice its breadth. What are the length and the breadth of the pool?

Solution: Let the breadth be x m. The length will (2x + 2) m.
Perimeter of the swimming pool = 2(l + b) = 154 m.
⇒ 2(2x + 2 + x) = 154
⇒ 2(3x + 2) = 154
⇒ 3x + 2 = 77 (Dividing both sides by 2)
⇒ 3x = 77 – 2 (Transposing 2 to RHS)
⇒ 3x = 75
⇒ **x = 25** (Dividing both sides by 3)
Hence breadth of the pool is 25 m.
Length of the pool = 2x + 2 = 2*25 + 2 = **52**

Solving Equations having variables on both sides

The equations having variables in both sides are solved similar to the above.

Example, an equation is $2x - 3 = x + 2$

Adding 3 both sides, $2x - 3 + 3 = x + 2 + 3$ or $2x = x + 5$

Subtracting x from both sides, $2x - x = x + 5 - x$ o**$x = 5$**

Example

Problem: Solve the following equations:
(i) $3x = 2x + 18$ (ii) $8x + 4 = 3(x-1) + 7$ (iii) $\frac{2x}{3} + 1 = \frac{7x}{15} + 3$ (iv) $2y + \frac{5}{3} = \frac{26}{3} - y$

Solution:

(i) $3x = 2x + 18$
$\Rightarrow 3x - 2x = 2x + 18 - 2x$ (Subtracting 2x from both sides)
$\Rightarrow x = 18$

(iii) $\frac{2x}{3} + 1 = \frac{7x}{15} + 3$
$\Rightarrow \frac{2x}{3} - \frac{7x}{15} = 3 - 1$ (Transposing $\frac{7x}{15}$ to LHS and 1 to RHS)
$\Rightarrow \frac{10x - 7x}{15} = 2$
$\Rightarrow \frac{x}{5} = 2$
$\Rightarrow x = 10$ (Multiplying both sides by 5)

(ii) $8x + 4 = 3(x-1) + 7$
$\Rightarrow 8x = 3x - 3 + 7 - 4$ (Subtracting 4 from both sides)
$\Rightarrow 5x = 3x + 0 - 3x$ (Subtracting 3x from both sides)
$\Rightarrow 2x = 0$
$\Rightarrow x = 0$ (Dividing both sides by 2)

(iv) $2y + \frac{5}{3} = \frac{26}{3} - y$
$\Rightarrow 2y + y = \frac{26}{3} - \frac{5}{3}$ (Transposing $\frac{5}{3}$ to RHS and $-y$ to LHS)
$\Rightarrow 3y = \frac{21}{3} = 7$
$\Rightarrow y = \frac{7}{3}$ (Dividing both sides by 3)

Example

Problem: A positive number is 5 times another number. If 21 is added to both the numbers, then one of the new numbers becomes twice the other new number. What are the numbers?

Solution: Let the number be x and 5x. Therefore,
$21 + 5x = 2(x + 21)$
$\Rightarrow 21 + 5x = 2x + 42$
$\Rightarrow 5x - 2x = 42 - 21$ (Transposing 2x to LHS and 21 to RHS)
$\Rightarrow 3x = 21$
$\Rightarrow x = 7$ (Dividing both sides by 3)

Hence the two numbers are 7 and 7*5 = 35.

Reducing equations to simpler form

Equations can be reduced to simpler form by

- Removing the denominators of both the sides.
- Opening the brackets.

Example, an equation is (6x + 1)/3 + 1 = (x - 3)/6

Removing the denominators by multiplying both sides by 6 because it is the LCM of the denominators of both sides,

6*(6x + 1)/3) + 1*6 = ((x - 3)/6)*6

- 2(6x + 1) + 6 = x - 3

Opening the brackets,

- 12x + 2 + 6 = x - 3
- 12x + 8 = x - 3

Adding 3 both sides,

- 12x + 8 + 3 = x - 3 + 3
- 12x + 11 = x

Transposing 11 to RHS and x to LHS,

- 12x - x = -11
- 11x = -11

Dividing both sides by 11,

- x = -1
- Cross-Multiplication

Example, an equation is (x + 1)/(2x + 3) = 3/8

By cross-multiplication, the denominator of LHS gets multiplied with numerator of RHS and vice-versa,

So, 8 * (x + 1) = 3 * (2x + 3)

- 8x + 8 = 6x + 9 (opening the brackets)
- 8x - 6x = 9 - 8 (transposing 6x and 8 other sides)
- 2x = 1
- x = 1/2 (Dividing both sides by 2)

Example

Problem: Solve the following linear equations:

(i) $\frac{x}{2} - \frac{1}{5} = \frac{x}{3} + \frac{1}{4}$ (ii) $\frac{3t-2}{4} - \frac{2t+3}{3} = \frac{2}{3} - t$

Solution:

(i) $\frac{x}{2} - \frac{1}{5} = \frac{x}{3} + \frac{1}{4}$

LCM of the denominators 2,5,3,4 is 60.

Hence multiplying both sides by 60.

$60(\frac{x}{2} - \frac{1}{5}) = 60(\frac{x}{3} + \frac{1}{4})$

$\Rightarrow$ 30x – 12 = 20x + 15 (opening the bracket)

$\Rightarrow$ 30x – 20x = 15 + 12 (transposing 20x and 12 to other sides)

$\Rightarrow$ 10x = 27

$\Rightarrow$ $x = \frac{27}{10}$ (Dividing both sides by 10)

(ii) $\frac{3t-2}{4} - \frac{2t+3}{3} = \frac{2}{3} - t$

LCM of the denominators 3,4 is 12.

Hence multiplying both sides by 12.

$12(\frac{3t-2}{4} - \frac{2t+3}{3}) = 12(\frac{2}{3} - t)$

$\Rightarrow$ 3(3t-2) – 4(2t+3) = 8 – 12t

$\Rightarrow$ 9t – 6 – 8t – 12 = 8 – 12t (opening the brackets)

$\Rightarrow$ t + 12t = 8 + 18 (transposing -12t and -18 to other sides)

$\Rightarrow$ 13t = 26

$\Rightarrow$ t = 2 (Dividing both sides by 13)